The Arithmetic Teacher

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THE ARITHMETIC TEACHER

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TABLE OF CONTENTS

		Page
Meaning and Skill—Maintaining the Balance	*William A. Brownell*	129
Arithmetic in New Zealand	*E. R. Duncan*	137
Arithmetic—An Old Subject in New Apparel	*Marie S. Ford*	143
Using Hand-operated Computing Machines in Learning Arithmetic	*Howard F. Fehr, George McMeen, and Max Sobel*	145
A Teacher Plans Her Day	*Mary E. Albrecht*	151
Arithmetic at the School Camp	*O. L. Davis, Jr.*	157
Arithmetic via Television		
I. A Report of the Pittsburgh Experiment	*Helen K. Strueve*	162
II. A Course for Teachers and Parents	*Irvin Brune*	165
III. Demonstration Lessons at Springfield	*R. C. Glazier*	168
Minutes of the Annual Business Session		168
The Controversy Regarding the Teaching of Higher-Decade Addition	*Frances Flourney*	170
Book Reviews		174

THE ARITHMETIC TEACHER

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Meaning and Skill—Maintaining the Balance*

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THE SUBJECT GIVEN ME poses a question. It is no academic question arising out of purely theoretical considerations. It is assumed that both meaning and computational competence are proper ends of instruction in arithmetic. It is implied that somehow or other both ends are not always achieved and that there is evidence that this is so.

Indeed there is such evidence. More than one school system has embarked upon a program of so-called meaningful arithmetic, only to discover that on standardized tests of computation and "problem solving" pupils do none too well. In such schools officials and teachers are likely to believe that they have made a bad bargain; and school patrons are likely to support them vehemently in this belief.

We may try to convince all concerned that the instruments used to evaluate learning are inappropriate, or at least imperfect and incomplete. True, standardized tests rarely if ever provide means to assess understanding of arithmetical ideas and procedures. Hence, the program of meaningful instruction, even if well managed, has no chance to reveal directly and explicitly its contribution to this aspect of learning. On the other hand, can we deny that the learning outcomes that *are* measured are of no significance? To do so is to say in effect that computational skill is of negligible importance, and we can hardly justify this position.

Why is there now the necessity to talk about establishing and maintaining the desirable kind of balance between meaning on the one hand and computational competence on the other?

Sources of the Dilemma

1. INCOMPLETE EXPOSITION OF "MEANINGFUL ARITHMETIC"

Perhaps those of us who have advocated meaningful learning in arithmetic are at fault. In objecting to the drill conception of the subject prevalent not so long ago, we may have failed to point out that practice for proficiency in skills has its place, too. It is questionable whether any who have spoken for meaningful instruction ever proposed that children be allowed to leave our schools unable to compute accurately, quickly, and confidently. I am sure that all, if asked, would have rejected this notion completely. But we may not have *said* so, or said so often enough or vigorously enough. Our comparative silence on this score may easily have been misconstrued to imply indifference about proficiency in computation.

If this has actually happened, we can scarcely blame classroom teachers if they have neglected computational skill as a learning outcome. It is characteristic of educational movements to behave pen-

* Paper read before the Elementary Section of the N.C.T.M. at the Milwaukee meeting of April 14, 1956.

dulum-wise. When we correct, we tend to over-correct. Just this sort of thing seems to have happened in the teaching of arithmetic. In fleeing from over-reliance on one kind of practice, we may have fled too far. It is a curious state of affairs that those of use who deplored the limitations of this kind of practice must now speak out in its behalf and stress its positive usefulness.

2. MISUNDERSTOOD LEARNING THEORY

A second possible explanation for our dilemma may be found, as is frequently the case in the practical business of education, in misinterpretations or misapplications of psychological theories of learning. Over-simplification of certain generalizations in learning theories as widely apart as are those of conditioning and of field theory could lead, and may have led, to the lessening of emphasis on practice in arithmetic.

a. Conditioning theory. According to the learning theory of one influential exponent of conditioning, once a response has been made to a stimulus a connection has been established. Thus, the child who says "Seven" in replying to the question, "How many are two and five?", sets up a connection between 2 and 5 as stimulus and 7 as response. If this is so—if the connection is actually formed by the one response—then at first glance practice might seem to be utterly purposeless.

But this inference is quite unfounded, as the psychologist in question makes abundantly clear. In saying that a connection is "established" he means no more than that a new neuromuscular pattern is available. He does *not* mean that our hypothetical child after the one experience will always, only, and instantly respond with "seven" when asked the sum of 2 and 5. If the response "seven" is to be the invariable one, moreover if the response is to be made in situations differing ever so slightly from the original one, then practice is required. In this theory of learning as conditioning, therefore, there is no comfort for those who would abandon practice as a means to promote the learning of arithmetical facts and skills.

b Field theory. No more comfort is to be found in field theory of learning. It is often said that one experience of "insight" or "hindsight"—before, during, or after success—is enough; but enough for *what?* It may be all that is needed to understand a situation and the method of dealing with it. Yet, if is one thing to know the general rationale for solving a complicated mechanical puzzle but quite another thing to be able to manipulate the parts correctly with facility, ease, and speed.

So in arithmetic it is one thing to comprehend the mathematical principles governing decomposition in subtraction—something that can come from a single insightful experience—but another thing to be able to subtract quickly and correctly. Indeed if in examples like $73-47$ and $52-19$ a child who possesses this understanding always thinks through the complete logical explanation, his performance will be impaired, at least from the standpoint of speed. Understanding and skill are not identical. A single instance of insight may lead to understanding but will hardly produce skill. For skill, practice is necessary.

3. INFLUENCE OF GENERAL EDUCATIONAL THEORY

A third explanation for failure to stress computational competence is to be found in the recent history of educational theory. It must be remembered that the place of meaning and understanding in arithmetic has been generally recognized for not more than fifteen or twenty years. In 1935 we were still under the influence of somewhat sentimental and unrealistic notions both about children and about the course of their development. In extreme form these notions led to a kind of teaching that was anything but systematic. Indeed, what children learned, when they learned it, and how they learned it was left pretty much

to the children themselves. Attempts to guide and direct learning and to organize learning experiences were frowned upon as "violating child nature" and as almost certainly productive of serious derangements of child personality. To those who held these views practice was anathema.

In the public schools, as contrasted with college departments of Education, this conception of the processes of learning and teaching did not gain much of a foothold. Nevertheless, it was in this climate of thought that stress on meaning in arithmetic put in its appearance. Those who were committed to the educational theory I have mentioned welcomed the new emphasis as confirming both what they did and what they did not do. If, deliberately or unwittingly, they accepted only the part of the emerging view most congenial to them, they committed an error that is very common and that is altogether human. Be that as it may, the consequences were none too good. True, these teachers may have been largely responsible for the quick and general endorsement of one aspect of meaningful arithmetic—learning with understanding. On the other hand, they could not themselves absorb the whole of it, and they remained hostile to practice in learning. Pupils taught by such teachers cannot be expected to make high scores on standardized arithmetic tests of skill in computation and "problem solving."

4. Inadequate Instruction on Meanings

I have suggested three hypotheses as explaining why we may not be obtaining balance between understanding and computational competence in arithmetic, on the assumption that our shortcomings relate to the latter (computational competence) rather than to the former. These three are: the possible failure of advocates of meaningful arithmetic to emphasize sufficiently the importance of practice in acquiring arithmetical skills; misinterpretations of psychological theories of learning which have had the effect of minimizing the place of practice; and the unwillingness of some teachers, who believe completely that arithmetic must be made intelligible to children, to provide the practice necessary for computational proficiency. May I add a fourth? It is that we may not as yet be doing a very good job in teaching arithmetical meanings as they should be taught.

There is ample evidence in psychological research on learning that the effects of understanding are cumulative. There is also ample evidence, if not in arithmetic, then in other types of learning, that the greater the degree of understanding, the less the amount of practice necessary to promote and to fix learning. If these truths are sound—and I think they are—then they should hold in the field of arithmetical learning. It follows that computational skills among school children would be greater than they are if we *really* taught them to understand what they learn

Again I remind you that meaningful arithmetic, as this phrase is commonly used, is a newcomer in educational thought. Many teachers, trained in instructional procedures suitable, say, to a view of arithmetic as a tool or a drill subject, find it difficult to comprehend fully what meaningful arithmetic is and what it implies for the direction of learning. Others than myself, I am sure, have, in conferences with teachers, been somewhat surprised to note that some of them are unfamiliar with major ideas in this conception and with methods of instruction adapted thereto.

Perhaps the commonest instructional error is, in a different context, the same one that has always distorted learning in arithmetic, namely, the acceptance of memorized responses in place of insistence upon understanding. Mathematical relationships, principles, and generalizations are couched in language. For example, the

relationship between a given set of addends and their sum is expressed verbally in some such way as: "The order of the numbers to be added does not change the sum." It is about as easy for a child to master this statement by rote memorization as to master the number fact, $8-7=1$, and the temptation is to be satisfied when children can repeat the words of the generalization *verbatim*. Similarly, the rationale of computation in examples such as: $33+48$ and $71-16$, makes use of concepts deriving from our number system and our notions of place value. But many a child glibly uses the language of "tens" and "ones" with no real comprehension of what he is saying. Such learning is a waste of time. To use an Irish bull, the meanings have no meaning.

I intend no criticism of teachers. Until recently there have been few professional books of high quality to set forth the mathematics of arithmetic and to describe the kind of instruction needed. Moreover, many teachers have had no access to these few books. Again, until recently not many courses of study and teachers' manuals for textbook series have been of much help. It is not strange, therefore, that though meaningful arithmetic is adopted in a given school system, not all members of the teaching staff are well equipped to teach it. As a result, their pupils, denied a full and intelligent treatment of arithmetic as a body of rational ideas and procedures, have been unable to bring to computation all the aid that could come through understanding.

Toward a Solution

So much for possible explanations—explanations of a general character—for our failure to keep meaning and skill in balance. I have no way of knowing the reality of any of the four hypotheses suggested or of the extent of its validity, to say nothing of the degree to which, taken together, the four account adequately for the situation. The fact remains that something needs to be done. What is the remedy?

Certainly we shall not get very far as long as we think of understanding and practice in absolute terms. I have deliberately done this so far in order to examine the issue in simple terms. Actually, it is erroneous to conceive of understanding as if it were either totally present or totally absent. Instead, there are degrees or levels of understanding. Likewise, not all forms of practice are alike. Rather, there are different types, and they have varying effects in learning.

LEVELS OF UNDERSTANDING

Consider the example $26+7$. The child who first lays out twenty-six separate objects, next seven more objects, and then determines the total by counting the objects one by one has a meaning for the operation. So has the child who counts silently, starting with 26. So has the child who breaks the computation into two steps, $26+4=30$ and $30+3=33$. So has the child who employs the principle of adding by endings,—$6+7=13$, so $26+7=33$. So has the child who, capable of all these types of procedure, nevertheless recognizes 33 at once as the sum of 26 and 7. All these children "understand," but their understandings may be said to represent different points in the learning curve. The counter is at the bottom, and the child who through understanding has habituated his response "thirty-three" so that it comes automatically is at the top of a series of progressively higher levels of performance.

All these levels of performance or of understanding are good, depending upon the stage of learning when they are used. For instance, finding the sum of 26 and 7 by counting objects is a perfectly proper way of meeting the demand at first; but it is not the kind of performance we want of a child in grade 4. At some time in that grade, or earlier, he should arrive at the stage when he can announce the sum

correctly, quickly, and confidently, with a maximum of understanding.

It is a mistake to believe that this last stage can be achieved at once, by command as it were. When a child is required to perform at a level higher than he has achieved, he can do only one of three things. (a) He can refuse to learn, and his refusal may take the form: "I won't," or "I can't," or "I don't care," the last named signifying frustration and indifference which we should seek to prevent at all costs. (b) Or, he can acquire such proficiency at the level he *has* attained that he will be credited for thinking at the level desired. Many children develop such expertness in silent counting that, in the absence of close observation and questioning, they are believed to have procedures much beyond those they do have. (c) Or, third, he may try to do what the teacher seems to want. If an immediate answer is apparently expected, he will supply one, by guessing or by recalling a memorized answer devoid of meaning. If he guesses, obviously he makes no progress at all in learning. If he memorizes, only unremitting practice will keep the association alive; and if he forgets, he is helpless or must drop back to a very immature level of performance such as counting objects or marks by 1's.

For the stage of performance we should aim for ultimately, as in the case of the simple number facts, higher-decade facts, and computational skills, we have no standard term. We may use the word "memorization" to refer to what a child does when he learns to say "Four and two are six" without understanding much about the numbers involved, about the process of addition, or about the idea of equivalence. If we employ "memorization" in this sense, then that word is inappropriate for the last step in the kind of learning we should foster. Hence for myself I have adopted the phrase "meaningful habituation." "Habituation" describes the almost automatic way in which the required response is invariably made; "meaningful" implies that the seemingly simple behavior has a firm basis in understanding. The particular word or phrase for this last step in meaningful learning is unimportant; but the *idea*, and its difference from "memorization," *are* important.

Teaching meaningfully consists in directing learning in such a way that children ascend, as it were, a stairway of levels of thinking arithmetically to the level of meaningful habituation in those aspects of arithmetic which should be thoroughly mastered, among them the basic computational skills. Too many pupils, even some supposedly taught through understanding, do not reach this last stage. Instead, they stop short thereof; and even if they are intelligent about what they do when they compute, they acquire little real proficiency. In instances of this kind both learning and teaching have been incomplete

How are teachers to know the status of their pupils with respect to progress toward meaningful habituation? Little accurate information is to be had from their written work, for both correct and incorrect answers can be obtained in many ways, and inference is dangerous. Insightful observation and pupils' oral reports volunteered or elicited through questioning are more fruitful sources of authentic data. Since children differ so much in their thought procedures, a good deal of this probing must be done individually. One of the most fruitful devices I can suggest for this probing consists in noting what children do in the presence of error.

I recall a conversation with a fourth grade girl whom I knew very well and who was having difficulty in learning—in her case, in memorizing—the multiplication facts. I asked her—her name was June—"How many are five times nine?" (This form of expression was used in her school instead of the better "How many are five nines?") Immediately she responded,

"Forty-five." When I shook my head and said, "No, forty-six," she was clearly upset. Her reply, after some hesitation, was, "No, it's forty-five." When I insisted that the correct product is 46, June said, "Well, that isn't the way I learned it." I suggested that perhaps she had learned the wrong answer. Her next statement was, "Well, that's what my teacher told me." This time I told her that she may have misunderstood her teacher or that her teacher was wrong. June was obviously puzzled; then she resorted to whispering the table, "One times nine is nine; two times nine is eighteen," and so on, until she reached "five times nine is forty-five." Again I shook my head and said, "Forty-six." When she was unable to reconcile my product with what she had become accustomed to say, I asked her, "June, have you no way of finding out whether forty-five or forty-six is the correct answer?" Her response was, "No, I just learned it as forty-five." Of course I did not leave her in her state of confusion; but the point of the illustration is, I hope, quite apparent: Her inability to deal with error was convincing evidence of the superficiality of her "learning" and of its worthlessness.

Compare my conversation with June with that I had with Anne, another fourth grade girl whom also I knew well and who, like the first girl, was learning the multiplication facts. When I asked Anne, "How many are five times nine?", the correct answer came at once, just as in June's case; but from here on, mark the difference. I introduced the error, saying that the product is 46, not 45. Anne looked at me in disgust and said, "Are you kidding?" I maintained my position that $5 \times 9 = 46$. Immediately she said, "Do you want me to prove it's forty-five?" I told her to go ahead if she thought she could. She answered, "Well, I can. Go to the blackboard." There I was instructed to write a column of five 9's and, not taking any chances with me, Anne told me to *count* the 9's to make sure I had 5. Next came the command, "Add them." When I deliberately made mistakes in addition, she corrected me, each time saying, "Do you want me to prove that, too?" Obviously, I had to arrive at a total of 45. Having done so, I said, "Oh, that's just a trick," to which she replied, "Do you want me to prove it another way?" Exposure to error held no terrors for this child; she did not become confused or fall back upon repetition of the multiplication table; nor did she cite her teacher as an authority. Instead, Anne had useful resources in the form of understandings that were quite lacking in the case of June whose discomfiture I have described.

Probing for understanding need not depend wholly on opportunities to work with individual children. On the contrary, there are possibilities also under conditions of group instruction when questions beginning with "How" and "Why" supplement the commoner questions starting with "What." The worth of valid knowledge concerning level of understanding is inestimable for the guidance of learning. The demands upon time are not inconsiderable, but no one should expect to get full knowledge concerning every pupil. The prospects are not hopeless if ingenuity is exercised and if the goal is set, not at 100% of knowledge, but more realistically at perhaps 20% more than is now ordinarily obtained.

TYPES OF PRACTICE

In crude terms, practice consists in doing the same thing over and over again. Actually, an individual never does the same thing twice, nor does he face the same situation twice, for the first reaction to a given situation alters both the organism and the situation. A changed being responds the second and the third time, and the situation is modified accordingly.

We must concede the truth of these facts. At the same time, for our purposes we may violate them a bit. Let us conceive

of practice of whatever kind as falling somewhere along a continuum. At the one end of the continuum is practice in which the learner tries as best he can to repeat just what he has been doing. At the other end is practice in which the learner modifies his attack in dealing with what is objectively the same situation (or what to him are similar situations). We may call these extremes "repetitive practice" and "varied practice," respectively.

An instance of repetitive practice is memorizing the serial order of number names by rote or applying the series in the enumeration of groups of objects. An instance of varied practice is the attempt, by trying different approaches, to find steadily better ways of computing in such examples as $43+39$, $75-38$, $136\div4$, and 32×48. Between the two extreme types of practice are innumerable others, differing by degree in the extent to which either repetition or variation is employed. But again, for our purposes, we may disregard all the intervening sorts of practice: we could not possibly name them all, or describe them, or show their special contributions of learning.

Both repetitive and varied practice affect learning, but in quite unlike ways. For illustration we may choose an instance of learning outside of arithmetic, for example, the motor activity of swimming. Suppose the beginner engages in repetitive practice: What does he do, and what will happen? Well, he will continue to use as nearly as he can exactly the movements he employed the first time he was in deep water, and the result will be that he may become highly proficient in making just those movements. He will hardly become an expert swimmer, but he will become an expert in doing what he does, whether it be swimming or not.

On the other hand, suppose that the beginner engages in varied practice. In this case he will seek to *avoid* doing precisely what he did at the outset. He will discard uneconomical movements; he will try out other movements, select those that are most promising, and seek a final coordination that makes him a good swimmer. Then what will he do? He will change to repetitive practice, for, having the effective combination of movements he wants, he will seek to perfect it in order to become more proficient in it.

The differences between repetitive practice and varied practice, both in what the learner does and in what his practice produces, are clearly discernible when we think of motor activities. They are less easily identified when we think of ideational learning tasks like the number facts and computational skills. But the differences are there none the less. The child who counts and only counts in dealing with examples like $36+37$ and $24+69$ is employing repetitive practice. The more he counts, the more expert he becomes in counting; but the counting will not, and cannot, move him to a higher level of understanding and of performance. In contrast is the child who, through self-discovery or through instruction, tries different ways to add in such examples. Under guidance he can be led to adopt higher and higher levels of procedures until he is ready for meaningful habituation. If then he does not himself fix his automatic method of adding, he can be led to do so through repetitive practice. In any case it is safer to provide the repetitive practice in order to increase proficiency and make it permanent.

The distinction between repetitive and varied practice, in their nature and in their consequence, is not always recognized in teaching. If repetitive practice is introduced too soon, before understanding has been achieved, the result, for one thing, may be blind effort and frustration on the part of the learner. Or, it may fix his performance at a low level, the level he has attained. No new and better procedure can emerge from repetitive practice though it may appear under conditions of drill when a child, tired of repetition or dis-

appointed in its result, abandons it in search of something new.

There may be an instructional error of another kind, one already alluded to. This error is to insist, to quote some, that "there is no place for drill in the modern conception of teaching." True, there is no place for unmotivated drill on ill-understood skills; but the statement goes too far in saying that there is no place at all for repetitive practice. How else, one may ask, is the final step of meaningful habituation to be made permanent; how else is real proficiency at this level of learning to be assured?

The kind of practice most beneficial at any time, then, is the kind best adapted to accomplish a given end. For illustration, let us return to June and Anne and their learning of the multiplication facts.

June was trying to master these facts by repetitive practice, by saying over and over and over again the special grouping of words for each separate fact. Her level of understanding of the numbers and relationships involved was close to zero. Unless engaged in almost ceaselessly, her repetition of verbalizations could give her little more than temporary control, and control, be it noted, of the verbalizations alone. Lapses of memory would be nearly fatal and would subject her to the hazards of guessing. In no way could her memorization of senseless phrases contribute much to sound learning of the facts themselves, not to mention the deficiences of its results for more advanced forms of computation and functional use. What June needed was not repetitive, but varied practice. By contrast, Anne, who was able to "prove" her announced products and who thereby demonstrated her full understanding of the relationship of the numbers, no longer needed varied practice and could safely and properly be encouraged to engage in repetitive practice.

We employ varied practice, then, if we wish the child to move upward from where he is toward meaningful habituation, and repetitive practice if we are endeavoring to produce true competence, economy, and permanence in this last stage of learning (or at any earlier stage which represents a type of performance of worth in itself). Practice has to be designed to fit the learner's needs, a fact which brings us back again to the individual and to the critical importance of accurate knowledge concerning his learning status.

In Conclusion

To sum up, the balance between meaning and skill has been upset, if indeed it ever was properly established. The reasons are many, some of them relating to educational theory in general, others to misconceptions of psychological theories of learning, others to failure to teach arithmetical meanings thoroughly, and still others to carry learning in the case of computation to the level I have denoted meaningful habituation, and then to fix learning at that level. I have discussed these matters at length, perhaps at unwarranted extent in view of the fact that the remedy for the situation can be stated briefly. The remedy I propose is as follows:

1. Accord to competence in computation its rightful place among the outcomes to be achieved through arithmetic;
2. Continue to teach essential arithmetical meanings, but make sure that these meanings are just that and that they contribute as they should to greater computational skill;
3. Base instruction on as complete data as are reasonably possible concerning the status of children as they progress toward meaningful habituation;
4. Hold repetitive practice to a minimum until this ultimate stage has been achieved; then provide it in sufficient amount to assure real mastery of skills, real competence in computing accurately, quickly, and confidently.

(Editor's Note on page 142)

Arithmetic in New Zealand

E. R. Duncan
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To understand New Zealand Arithmetic Programme it is first necessary to recall certain things about New Zealand. It is necessary to remember, for instance, that it is a small country with a population about a quarter that of the city of New York, spread over two islands whose combined length is a little over 1000 miles and whose average width is about 100 miles. It is necessary also to remember that it is a young country, strongly conscious of its historic links with Great Britain.

To serve its small but scattered population, New Zealand has a highly centralized school system. Its Department of Education exercises decisive control over teachers and curricula. The benefits of this centralization are what one might expect, a good standard of educational opportunity for all our children and the equitable and economic use of our resources. But its dangers are also foreseeable, the most significant for the curriculum being tendencies towards uniformity and conservatism. Despite the leadership of an enlightened Directorate and the efforts of Inspectors and other professional counsellors, these tendencies are likely to persist as long as the same Departmental publications, equipment, and syllabuses are used in all our 1900 schools, the 1100 small rural ones, as well as the large ones in the cities.

Since the first settlers did their best to transplant into the new land the institutions of the land they had left, the English tradition is strong in New Zealand education. But as the country has grown and overcome the worst effects of its geographical isolation, it has thrown off much of its early dependence on English traditions and been free to accept other influences. European and American writers have shaped our thoughts in more than one field, but we have not been content merely to accept and imitate, and a resourceful independence in educational thought has developed with the application of original thinking to our problems, the adaptation of ideas through experience and research, and the growing confidence of a maturing people.

An Evolving Arithmetic Programme

The New Zealand Arithmetic Programme has for a number of reasons been rather less liberal than that of other subjects of the curriculum. In any country it is true that the nature of Arithmetic all too often makes it a target for the misapplied conscientiousness of the teacher whose desire for determinable results reduces the subject to an arid formalism, and New Zealanders have not been free from this temptation. Indeed, there are reasons why teachers here have been more exposed to it than teachers in many other countries. For one thing New Zealand has hardly had time to break free from the severely practical outlook of the early settlers with their payment of teachers by results and their idolatry of the academic aims of the English public school. For such reasons writers on Arithmetic, notably Thorndike and Ballard, have met receptive readers here and drill methods became firmly established in our schools.

But Arithmetic could not escape the liberalism of the period between the wars. In 1934 the elementary school leaving examination in this country was abolished

and the schools were set free to develop their individual methods and programmes. By the outbreak of the second world war a general revision of the common curriculum was begun in the spirit of the new freedom and Arithmetic was ready to be included in the revision. The year 1943 marks an important stage in development of New Zealand Arithmetic for it was in this year that the revision was completed and the new syllabus was published, together with a manual for teachers of Infant Rooms (first two years of school) and a series of textbooks to be supplied free to all children in the other classes. Consequently any description of current practice can fairly begin at this point.

Arithmetic in the Infant Room

New Zealand children begin their schooling at the age of 5 and pass their first two years in what we call the Infant Room. At this stage they spend about 20 minutes each day in finding out about numbers by various activities which follow a pattern like that set out in such books as Brideoake and Grove's "Arithmetic in Action." To help provide number experiences, the Department has issued a large quantity of equipment, mainly counters, measuring sticks, balances and form-boards as well as jig-saws, mosaics and similar apparatus primarily intended for the free activity period with which most of our Infant Rooms begin their day. Teachers also make a great deal of equipment to extend the range of activities for their own class.

A manual giving suggestions on how to put the new syllabus into practice was supplied to all teachers, but no provision was made for workbooks, for children and, apart from some commercially prepared books of limited popularity, this gap has not yet been filled. Because of the Department's policy of issuing all books without direct cost to the child, it seems unlikely that expendable material such as workbooks will be available here unless some future private venture turns out to be more successful than those up to now.

Although the number programmes in our Infant Rooms are hardly as well developed as our programmes in Reading, the work being done in number is generally developmental and forward-looking. Progressions have not always been clearly established but, everything considered, number in our Infant Rooms has more purpose and meaning than in the classes to follow.

Arithmetic in the Standard Classes

At the age of seven the child transfers from the Infant Room to the first of the six "Standard" classes which constitute our primary school (starting at age 7). This transfer should be only a nominal one but in many of our schools it has come to have considerable significance in that it is taken as the point at which the free approach of the Infant Room is suddenly exchanged for a more formal one and this formalism once established is allowed to become a habit throughout the primary school.

A sudden change in teaching methods at the time of the transfer to Standard 1 is particularly common in Arithmetic. This is due partly to the Department's policy of issuing a teacher's manual and experiential equipment for the Infant Room and then issuing textbooks with no manulas or equipment for Standard 1. It is also partly due to the shift from the Infant Room syllabus which merely suggests a series of activities to the Standard 1 programme of learnings which many teachers feel they have to complete as systematically as possible.

There is little to justify this move to formalism, for although the syllabus for the Standard classes is precise in outline, it is liberal in spirit. It has its closest link with certain Scottish work plans but is similar in many respects to American Arithmetic Programmes issued about the same time.

Important learnings at successive stages may be summarized as follows for Standard Classes (age 7+):

Year 1—Addition facts to 20.
Year 2—Multiplication facts, compound addition and subtraction.
Year 3—Long multiplication and the four rules in money.
Year 4—Long division and extensions of the four rules.
Year 5—Fractions.
Year 6—Extensions of fractions.

The fraction concept is introduced from the first year but decimals are not introduced until the fourth year Standard Class due to our English coinage which is not on a decimal system. Unlike most American programmes, we tend to drop shopping after the second year Standard Class and recommend the use of Ready Reckoners after the first year.

Social Arithmetic is treated more as a separate section of the syllabus and not woven into its fabric as seems to be done in America The main learnings might be summarized like this:

First two years—shopping and measuring
Next two years—weights and measures, time, and travel.
Last two years—more general buying and selling, taxes, etc.

The 1943 Textbooks

To put the syllabus for Standard Classes into practice the Department issued a series of textbooks. Before that time Arithmetic texts in this country had been supplied by private enterprise and consisted mainly of sets of formal exercises to cover the topics of the course. But with its texts the Department attempted to provide for a better understanding of each process and its relation to other processes and activities, and there is no doubt that the use of these texts throughout the country has done much to improve our teaching of Arithmetic.

From the outset, however, these books had to face unusual difficulties for not only were they the first in this country to include more meaningful approaches with material which was not always directly connected with computation, but they also had to try to meet the demands of widely different situations, from large city schools with single-unit classes to small rural schools with all or most of the classes working under one teacher in the same classroom. In addition, it was hardly possible to provide material within the space limits of texts of about 200 pages to suit the full range of individual differences in each class. So for a variety of reasons and in many different ways the texts had to be content with a middle course.

As a further handicap teachers, had to use textbooks without manuals or pupils' workbooks. The lack of teachers' manuals meant that some approaches were not fully understood and that in some ways the liberalism of the revision was not properly appreciated. The lack of pupils' workbooks means that a good deal of unnecessary waste of time is involved in the pupils' copying examples; also that an opportunity is lost for the presentation of alternative exercises, algorisms and research projects for which there is not enough space in the textbooks.

From the beginning it was obvious that the text books would need to be revised quite soon. For one thing, it was realized that they would have to be modified to suit the changed conditions which they themselves had created. Accordingly, no sooner were the books in use than the machinery was established for evaluating and revising them. Committees of teachers and inspectors were set up in different parts of the country to collect opinions and research data, and by 1951, when this work had gone far enough, the actual revision of the books began.

The detailed study of textbooks has

shown many things not only about the books themselves but also about the teaching of Arithmetic generally in this country. The results of this study are being incorporated in the revised textbooks which are now being prepared and issued to all schools, and in the new syllabus which is being worked on in the light of the research we have been able to do. Many of the problems we face will be far from new to American readers. Because of our unique background, our solutions and the principles we will develop will be quite different from those already well established in the States.

Emergent Problems

One of our most important findings is that, particularly in the upper part of our schools, too many of our teachers are still thinking of Arithmetic as mechanical computation with habit more important than understanding. Of the hundreds of teacher suggestions and criticisms received during the revision of our Arithmetic textbook far too many were concerned with superficial drills and too few with meaning and motive. Many teachers said that the explanatory material in the 1943 texts was a waste of space. They complained that such material took up space which could be better given to further mechanical exercises. Attempts to introduce rationalizations were met in some quarters with the comment that the pupils did not need to understand, so long as they could do the sum, for instance.

Failure to appreciate the importance of understanding is clearly shown in the teaching of compound subtraction. Despite the fact that, right or wrong, the 1943 syllabus recommended the "equal additions" method and our textbooks were geared to it, at least 60% of our teachers (my estimate) still use a method which is quite incomprehensible to themselves and to their children. This method is a curious adaptation of "decomposition" and, so far as I know, is peculiar to New Zealand in the way it is practiced here. It consists in "borrowing" from the minuend and "paying back" to the subtrahend. When I have brought it to the attention of children, they have quickly realized that it is nonsense and just as quickly originated a better method themselves or understood one which is more intelligible. So far as I can see, the only reason why some of our teachers use such an outmoded method is that it was the one by which they themselves learned to subtract and it has never occurred to them that they did not know what they were doing.

In teaching the four rules (the fundamental processes), for example, many of our teachers give scant attention to the underlying process with the result that even the teacher is unaware of the different kinds of subtraction or division which are involved in either process. Similarly in evaluating pupil progress too many of our teachers are satisfied with tests which measure mechanics only and are not sufficiently concerned with finding out the nature of their pupils' understanding.

There is also confusion in the minds of some of our teachers about Social Arithmetic. They do not distinguish between mathematical meaning and social significance. I know many teachers who feel that giving meaning to Arithmetic signifies placing it against its social background and I have frequently seen shopping and measuring activities treated as separate learnings instead of being related to fundamental understanding. Very few of our teachers regularly use a social problem before presenting a new process; almost invariably it is brought in as a test of their pupils' ability to apply the new skill.

In the same way the grading of examples in sequence is allowed in some schools to become nothing better than a thinly disguised form of drill. The true nature of a process is often lost sight of. I know of one teacher who had reduced—or is it increased?—the difficulties in subtraction to 47 types, on the principle that

once you have led your pupils through these types, you have taught them how to subtract! Quite recently the Department issued a series of diagnostic tests under the title "Easy Steps in Arithmetic" but many teachers seized on the tests as a tortuous path which all their pupils must tread and completely destroyed the remedial value of the publication.

The Meaning Theory is being applied with some success in the child's first three years in our schools (ages 5–8) but beyond that point even some of our best teachers are not sufficiently aware of the implications and practical effects of the theory. Not that we are unfamiliar with such writers as Brueckner and Grossnickle, Clark and Eads, Spitzer, Wheat, Wilson, and other Americans. But so far they have had less practical application in our schools than American writers such as Fernald, Mursell, or Strickland.

The initial training period at Teachers College, In-service Training Courses, Departmental Inspectors, our professional magazines, and the increasing supply of literature on the methodology of Arithmetic—all are being used to make our teachers better acquainted with meaningful practices. In addition, we hope to provide all teachers with manuals which will not only serve to explain the approaches in the revised texts but will also discuss the broader implications of these ideas and give suggestions for alternatives.

The revised Arithmetic texts themselves are being used directly to train teachers. Vocalizations and rationalizations in the texts are included to help the child but they may also give the teacher fresh insight. The books are being modified in many ways. The number of purely mechanical exercises is being reduced to make way for developmental content but the actual amount of practice is maintained by the introduction of condensed material, devices, games, and activities which pupils will be able to use repeatedly and for different purposes. The books will provide many more opportunities for discussion, research, and activity.

We have tried out new approaches in a large number of schools before incorporating them in the new books. For instance, we have had particular difficulty with compound subtraction because of the survival of an irrational method, as mentioned before. In presenting "equal additions," we now begin with a large number of practical and discussion exercises to establish the necessary understandings before any written examples are attempted. These steps were worked out in different centres throughout the country and we are confident that they will improve the early attack on "equal additions," at which point the method is generally thought to be inferior to "decomposition."

Our research has not always given us such happy results as we feel we are having with compound subtraction. Rationalizing division of fractions, for example, has not reached a point at which we can feel assured of success and this makes us aware of the limitations of our research facilities. Because of their smallness, our University Schools of Education cannot undertake major research projects of this kind. Although the New Zealand Council for Educational Research has done some very fine original work, its organization and commitments would hardly allow it to collaborate on such tasks. Consequently we are forced to rely on the co-operation of already burdened teachers and inspectors.

Despite our limitations we have a number of research projects in progress, two of which are described as follows:

1. *Providing richer opportunities for more able children.* Our present solution is only a partial one and is limited to helping the more able youngsters in the last two years of their elementary school life. We are at present working on an additional book in our series to provide suitable

material for these children. The material to be included in this book must satisfy two criteria—it must be developmental mathematical training, and it must be such that it will not be immediately repeated at our high schools. It is an interesting task and our results so far are encouraging.

2. *Arithmetic for Infant Rooms.* At this level we are studying the effect of deferring counting on growth in number concepts. We are experimenting with pattern apparatus of the Stern and Cuisenaire type. Although it is too soon to give a firm opinion, present indications are that some approach similar to Cuisenaire's will be adopted here.

The Outlook for New Zealand Arithmetic

The high degree of centralization in the New Zealand education system may lead to too much uniformity, as I mentioned earlier, but it also can help in the development of teaching practices. For this reason Arithmetic in New Zealand can be expected to move towards a more satisfying expression of the Meaning Theory. There have been rapid advances over the last few years and the general spirit of our elementary schools must give impetus to this forward move. Consequently, although I may have appeared in this article to be critical of our handling of this subject, I should finish by saying that I am optimistic for the future and am confident that Arithmetic in our schools will be an increasingly satisfying and lively experience for our children.

EDITOR'S NOTE. Mr. Duncan is an officer in the New Zealand Department of Education. This year he is a visiting instructor at the teachers college at Jersey City, New Jersey where he is working with Dr. Foster Grossnickle. He has been much concerned with the problems of teaching and learning arithmetic in his native country and in comparing the work there with his impressions of arithmetic in the United States. The opinions he expresses in this article are personal and do not represent the opinions of the department in which he normally works. It will be interesting to learn how his appraisal of our work changes as he has opportunity to observe. Teachers in this country have not solved as many educational problems as Mr. Duncan infers. It is refreshing to learn that others are concerned with making arithmetic an experience that has meaning and significance instead of merely a body of facts to be memorized. Perhaps we will have opportunity to hear Mr. Duncan on some of our National Council programs.

Brownell—Meaning and Skill

(*Continued from page 136*)

EDITOR'S NOTE. Dr. Brownell is talking particularly to teachers in school systems that have initiated programs of meaningful arithmetic but his message is for all teachers. He indicates that it is difficult to comprehend fully what meaningful arithmetic is. He warns against teaching for memorized generalizations under the guise of teaching meanings. A good sound program of arithmetic will develop meanings and understandings as basic and necessary to a functional program which does not slight computation and problem solving. In this country we study arithmetic primarily for its contribution to intelligent and successful living in social, economic, and cultural situations. We do not thereby neglect those mathematical elements which give substance to arithmetic as a science, We are not primarily concerned with arithmetic as a puzzle for the amusement of the few. In our society the abilities to read and to form mathematical conclusions are very important.

Marguerite Brydegaard Becomes Associate Editor

Beginning with this issue of THE ARITHMETIC TEACHER, Mrs. Marguerite Brydegaard of San Diego State College, San Diego, California joins the staff as a new associate editor. Mrs. Brydegaard replaces Professor Esther Swenson of The University of Alabama whose other commitments became so heavy that she had to withdraw from the magazine. It is a pleasure to welcome Mrs. Brydegaard who has been an active worker in the field of arithmetic for a number of years. She brings a rich experience of teaching, supervision, and study and writing to her new extra job. People in the western area may send manuscripts and ideas directly to Mrs. Brydegaard.

Arithmetic—An Old Subject in New Apparel

MARIE S. FORD
Geneo Central School, Geneo, N. Y.

THIS IS AN S.O.S. to all teachers and would-be teachers of arithmetic. Your help is needed. May we enlist you as a recruit? The purpose of this distress call is to save our youth from being exposed to the limitations and barriers set up because of our own experiences with the study of arithmetic.

First of all, I want to explain that I am a classroom teacher of many years' experience. This Spring when I planned my summer course, I included a Seminar in the teaching of arithmetic. While discussing my choice with my principal he asked, "You've been successful in your teaching of arithmetic, haven't you? Do you feel the need of such a course?"

I replied that I considered I had been fairly successful, but a course revealing up-to-date methods might be helpful. But I really thought, "What *new* things could be done in such an old and tried field as mathematics?"

In order that others may be converted to this new and different way of presenting mathematics, I am here relating my experience of what took place when I was brought "face to face" with "meaningful mathematics." Someone has to be the "guinea pig" so, it might as well be I.

The first day that the summer school class convened everything went well, and our course was set. But the second day, and the third, our ship began to quaver in the storm of criticism. When it was shown that we had missed the real meaning and relationships of our number system, our self-esteem was pretty low. I felt baffled, bewildered, a little angry and quite upset. It was like being exposed to the measles and there was nothing I could do about it.

Everywhere I went, I saw and heard people talking "math," in a little huddle here, a couple over there, at lunch, in a greeting to each other. Even those who were not in the class were asking questions. The whole campus was in quite a state of furor.

I found myself "talking math," "thinking math," "dreaming math." I was caught in a whirlpool of confused ideas. All this was good, although I didn't realize it at the time. This is the way "meaningful math" takes hold. We have been dealing for so long with figures, numbers, rules, definitions, formulas—all having little or no meaning—that the new way is like a first try at learning a foreign language.

However, soon the light began to dawn. There was a faint ray of hope showing dimly above the horizon. Now, I am completely "sold" on this way of teaching. I realize that it means hours of study, training and preparation before I'll be equipped to present "meaningful mathematics."

Replacing ideas that we have fostered and nourished for so long will not be easy, but what *real* learning is easy? In these days of electronics, hydrogen bombs, rocket ships, proposed man-made satellites and space travel, dare we hinder the discovery and projection of new ideas by our young people? The future of our country is in the hands of our youth. In order that we may contribute to that future, through the youth of our classrooms, is the learning worth the effort?

Now to get down to "brass tacks" and the arithmetic teacher. In the simple words of a classroom teacher, the underlying principle for teaching "meaningful arithmetic" is progression from the known to the unknown. Begin with concrete

things. These will be utilized in mathematical experiences brought about in the classroom, and in the case of the pupils in the higher grades, often from some former experience, upon which the pupils may draw. The teacher brings about many learning situations in which the pupils are allowed to explore and to make their own discoveries, with the aid and guidance of the teacher, often using representative materials. In the lower grades, these materials might be beads, shells, discs, toothpicks, etc. In the upper grades, pupils will use lines, graphs, diagrams, geometric figures, and algebraic representations. Through the use of these materials, generalizations will be formed. These will be remembered because the pupil has discovered and figured them out for himself. We now progress to the abstract. We use drill, for those who need it, first to show relationships, then to "fix" a fact so that it sticks. Practice will be given to insure correct computation and accuracy.

Don't you primary grade teachers see that you are laying foundations which are to be built upon by other teachers farther along the way? Make sure, then, that the foundation is firm, and solid, and right, that it may never be necessary to tear down or to rebuild that foundation.

Shall we hold the children in darkness "behind the iron curtain" of rigid and conformed thinking or shall we liberate them into the sunlight of free thought? Don't you feel that it is your privilege and your duty to "open the gates" and teach arithmetic that has *real* meaning to the child?

"How do I do this?" you may ask. Take a course in modern methods for teaching arithmetic. Read the articles in current magazines on this subject. Look into and study the research that is being done in this field. Remove the blinders and open your eyes to the new and better trend in the mathematics area.

Will you join that great army of teachers? Your help is needed. What greater profession could you choose, than one in which you have the opportunity to inspire, aid, mould and direct the learning of our children? If this can be done through teaching in the field of mathematics, then let us teach "meaningful mathematics."

Science and mathematics go "hand in hand." Success in industry, engineering, executive positions, in most of the high places of our land, require a knowledge and clear understanding of mathematics. Are we giving our pupil this equipment, or in contrast, are we crippling him by causing him to rely upon cane, crutches, and artificial limbs? This was done to us. Will you in turn hold him down to your level and hinder his thinking with mathematical terms, symbols, and generalizations of which he has little understanding? It is high time that we realized what we are doing to these children, and that there is a new and better way to teach arithmetic. We are making progress in the field of reading, let us not fall behind in the field of mathematics. It takes planning, preparation, ingenuity, and cooperation to extend such a program throughout the school.

The world is waiting for that boy and that girl, who, unhampered, and uninhibited, can soar to unparalleled heights because no shackles have bound them. They have been taught "meaningful arithmetic" all along the way. Will they come out of your classroom?

EDITOR'S NOTE. Each year many teachers attend summer sessions in order to learn new ideas and develop new competencies. But is the much larger group who have had little or no stimulation and study for a number of years who probably need their complacency challenged? Mrs. Ford had an experience which first disturbed her and finally provided a new vision of the teaching and learning of arithmetic. We need many more Mrs. Fords.

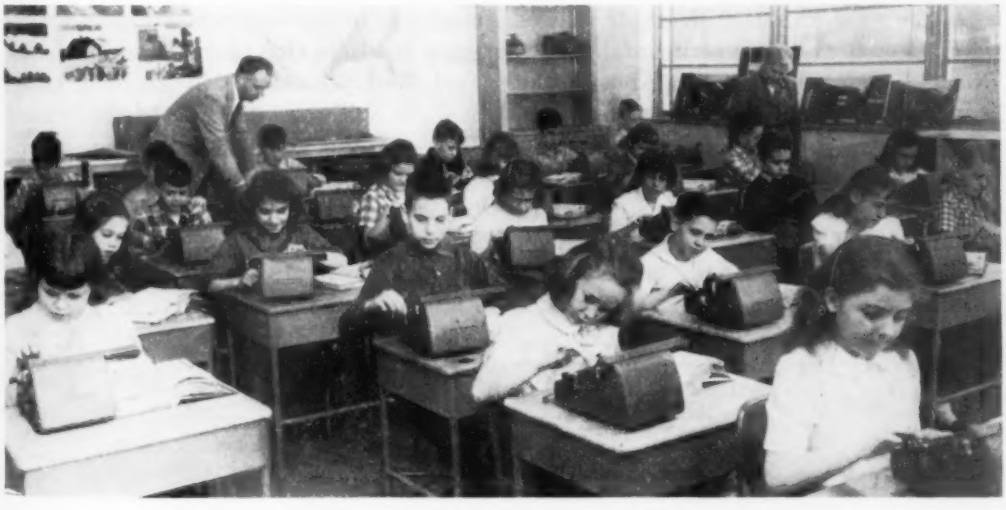
Using Hand-Operated Computing Machines in Learning Arithmetic

HOWARD F. FEHR, *Teachers College, Columbia University*
GEORGE MCMEEN, *State Teachers College, Newark, N. J.*
MAX SOBEL, *Southside High School, Newark, N. J.*

RECENT REPORTS OF EXPERIMENTS have announced rather startling claims concerning the great amount of arithmetic learned in a rather short period of time. These researches made use of certain materials and devices. Since such great improvement in learning has not hitherto been thought possible, and since frequently such reports appear to be "headlines" rather than scientific fact, there is need for controlled scientific experiment to judge the values of these claims. In general there has been no royal road to improved learning in the past, only slow, careful and hard won progress. The following is a report of an experiment, scientifically designed to measure the value of the use of computing machines in the learning of arithmetic.

In the Spring of 1955, a controlled experiment on learning multiplication by a two-digit multiplier was performed by the authors of this report. The experiment was designed to test whether or not the use of computing machines, hand operated, aid in learning both the meaning and understanding, as well as developing skill in paper and pencil computation. At the end of the *two week* experiment there was no significant difference in favor of either the control group (usual learning) or the experimental group (those using machines) either in computation or in reasoning ability in arithmetic. However, a number of concomitant factors indicated that in a prolonged use of machines there might be an advantage of using machines.

These factors may be listed as (1) the en



BOYS AND GIRLS WORK WITH MACHINES

joyment of the pupils who used machines in their arithmetic study, (2) the lack of techniques in using machines to teach arithmetic on the part of the teachers. They had to be taught not only how to use the machines, but how to teach by the use of machines. At the end of the experiment the teachers in the experimental group had become enthusiastic and wished to continue using the machines. (3) The experimental groups had to learn both machine computation and paper and pencil computation in the same length of time of study as the control groups which put all this time on paper and pencil computation. Thus the experimental group, despite the heavy additional learning task, made normal gain in paper and pencil work.

The Half-Year Experiment

To make a more satisfactory test of the value of machines in learning arithmetic, a half-year ($4\frac{1}{2}$ months) experiment was performed in the Fall and Winter of 1955–56. This experiment was designed to test only one hypothesis, namely:

> *Pupils who use computing machines to learn arithmetic will gain significantly in paper and pencil computations, and in arithmetic reasoning over those who do not use the computing machines.*

In one school, four fifth grade classes were selected as the experimental group. Two classrooms were equipped with the Monroe Educator Model hand-operated computing machine, so that each student had an individual machine. Two classes used the same room, one class studying in the morning, the other in the afternoon session. The teachers who taught these classes were instructed how to use the machine for all four operations and told how to use the machines in instructing in arithmetic. These teachers did all the teaching in the regular assigned daily period of 35 to 40 minutes. No other person instructed the classes, and no additional time was given to arithmetic instruction. A set of directions given to the teachers during the experiment is attached to this report (see Appendix I). The regular textbook and syllabus was followed without any deviation, and when the study of fractions appeared the machines were not used.

The control group was scattered in four communities where the same textbook and/or syllabus was used and where the fifth grade class was comparable in I.Q. and arithmetic achievement to the four experimental classes. Both the control and experimental groups were matched very well according to socio-economic backgrounds, i.e. types of communities, types of schools, types of occupation of parents, and quality of teachers. The textbooks used in all the classes were recently published and emphasized a meaningful approach to learning. These classes had the same length period of instruction each day, and used such materials and mechanical aids as they were accustomed to use in regular teaching. However, they did not use computing machines in any manner. All classes were given the Beta Otis Quick Scoring Mental Test, and pretested with Form J, Intermediate Arithmetic Stanford Achievement Test. At the conclusion of the experiment all classes were tested with Form K. Stanford Intermediate Arithmetic Achievement Test.

The following tables summarize the test statistics of the experiment.

TABLE I

Means of Control Groups

	1	2	3	4
I.Q.	110.68	110.52	107.83	107.70
PRETEST COMPUTATION	4.71	5.10	4.56	4.67
POST TEST COMPUTATION	5.47	5.32	5.30	5.96
GAIN IN COMPUTATION	*.76* (8 mo.)	*.22* (2 mo.)	*.74* (7 mo.)	*1.29* (13 mo.)[1]
PRETEST REASONING	5.80	5.79	5.84	5.40
POST TEST REASONING	6.18	5.98	6.25	6.30
GAIN IN REASONING	*.38* (4 mo.)	*.19* (2 mo.)	*.41* (4 mo.)	*.90* (9 mo.)[1]
N	23	21	22	26

(The above means are given in terms of grade norms.)

[1] Class 4 of the control group, upon investigation was found to have, in addition to the regular teacher, two full time practice teachers. These teachers gave extra instruction in decimal operations not prescribed for the experiment. There were a number of psychologically disturbed children in this class who contributed to the poor showing on the initial tests and were eliminated from the final test. In the Mean Gain by Groups, Table III, this class was therefore omitted.

TABLE II

Means of Experimental Groups

	1	2	3	4
I.Q.	107.18	112.50	111.74	107.84
PRETEST COMPUTATION	4.70	4.87	4.26	4.56
POST TEST COMPUTATION	5.57	5.55	5.48	5.32
GAINS IN COMPUTATION	*.87* (9 mo.)	*.68* (7 mo.)	**1.22* (12 mo.)	*.76* (8 mo.)
PRETEST REASONING	5.49	5.57	4.88	5.43
POST TEST REASONING	6.21	6.28	6.03	6.40
GAINS IN REASONING	*.72* (7 mo.)	*.71* (7 mo.)	*1.15* (12 mo.)	*.97* (10 mo.)
N	25	20	19	23

* 1 year = 10 mo. of school work. Hence 1.22 = 12 school months.

TABLE III

Mean Gains by Groups

	Reasoning		
	Pretest	Post test	Gain
CONTROL (N = 92)	5.81	6.14	.33
EXPERIMENTAL (N = 87)	5.46	6.23	.77
	Computation		
	Pretest	Post test	Gain
CONTROL (N = 92)	4.79	5.36	.57
EXPERIMENTAL (N = 87)	4.61	5.48	.87

We thus see that the experimental group gained 4.4 more months in reasoning and 3 months more in computation than the control group. It is true that the experimental group began at a lower point in both reasoning and computation, but the gains for a period of $4\frac{1}{2}$ months are greatly in favor of the experimental group.

Looking at the results of the experimental group only we can say that in a period of $4\frac{1}{2}$ months, these classes made gains of 9, 7, 12, and 8 school months of arithmetic learning in computation and 7, 7, 12, and 10 months in reasoning. This is to be compared with corresponding gains of 8, 2, and 7 months in computation and 4, 2, and 4 months in reasoning on the part of the control group. The control group made normal expected gains, but the experimental group exceeded normal expectancy.

As a further means of evidence, 25 matched pairs of pupils were selected at random to meet the criteria (1) I.Q. within 5 points of each other and (2) pretest scores not to differ by more than .3. The following table summarizes the results.

This is a statistically significant gain in computation in favor of the experimental group at the .95 level of confidence. All the evidence therefore points in favor of sustaining the hypothesis.

Summary

The experiment showed a significant *gain* in both computation and reasoning on the part of the experimental group. There was not, however, statistically significant difference in final achievement standing between the two groups at the end of the experiment. It is to be regretted that the experiment could not run for one full year, but all indications are that the group using machines would excel.

The gains for the most part were not phenomenal, but within a level of reasonable expectancy. It must also be noted that the experimental group had to learn a double task within the same teaching time as allotted to the control group. They learned how to do machine calculation as well as paper and pencil calculation, and made superior gains in arithmetic both in reasoning and computation.

More significant is the attitude and responses of both the teachers and the pupils in the experimental group. They enjoyed their learning, they gained a better understanding of arithmetic, there was an *esprit de corp* in the machine classes not prevalent in ordinary classes. The values attributed to the use of the machines are recorded in the frank letters from these experimental teachers. These letters are attached as Appendix II.

In conclusion it is safe to say that, (1) Machine-taught students gain more in reasoning ability; (2) Machine-taught students gain more in computation ability; (3) Machine-taught students learn more also, because they understand machine computation as well as ordinary arithme-

TABLE IV

Mean Scores of 25 Matched Pairs

	Pretest		Post test		I.Q.
	Reasoning	Computation	Reasoning	Computation	
EXPERIMENTAL	5.57	4.67	6.27	5.61	107.92
CONTROL	5.56	4.69	6.01	5.30	107.56
GAINS OF EXP.			.70	.94	
GAINS OF CONTROL			.45	.61	
DIFFER IN FAVOR OF EXP. GROUP			.25	.33	

tic; (4) The interest of students and teachers in arithmetic is heightened by the use of machines; and (5) The machines fit into our present culture. They give additional learning.

Appendix I

Suggestions for using the computing machine to aid paper and pencil computation.

1. Be sure to use the machines continuously, at least 3 days and preferably 4 days each week. The quicker machine skill is obtained, the more it can be used to aid the understanding and developing of skill in computation.

2. The machines can be used to check paper computations. But, also do the reverse, that is, have the children compute by machine, and then check the result by paper and pencil.

3. In using the machines for column addition, have the children add the units' column first. Have them note the partial sum which shows the number of tens "carried." Then add the tens' column, then the hundreds' column. Thus the machine does just what the children do in their computation.

4. In subtraction, after the minuend is put in the machine, have the pupils subtract the unit digit of the subtrahend first. Have the pupils note how the minuend was changed, especially where the tens' digit is decreased by 1, due to exchange or borrowing. Then subtract the tens' digit, and so on. Thus again the machine does exactly what we do on paper with pencil.

5. In multiplication, stress the meaning of the shift of the carriage as multiplying by the tens' digit, then the hundreds' digit, and so on. Always try as much as possible to make machine and pencil and paper computation agree in method.

6. Do not insist on speed. However, insist on ease, sureness, security, knowing. This will have the greatest effect on becoming *correct* computers.

7. Always make the paper and pencil work a little harder than previous machine computation. This shows the value of the machine and also encourages paper and pencil learning before machine learning.

8. Don't forget to have children check by paper and pencil (also interchanging multiplier and multiplicand, multipying quotient by the divisor, etc.). They will not always have a machine to check.

9. Do not omit problems. They are as important as mere computation. However, use the machine to solve problems.

10. Be sure to think of new problems to interest pupils as they get to be expert with the machine. Finding an average is a good one, as it involves finding a sum and then dividing the answer.

Appendix II

"Our children have gained a better understanding of arithmetic through the use of the machines; our teachers have been spurred to critically evaluate their procedures in teaching arithmetic and in analyzing the material assigned to the grade level".

Principal,
Memorial School

"The stimulus of the machines created a challenge in the child not only to learn the machine operations, but also to show that his own written arithmetic would be as correct as the work he did on the machine.

The class developed an *esprit de corps* about using the machines. The quick students helped the slower ones and everyone seemed eager to maintain a high class average.

At this age level the machine was a good disciplinary device. The children not only looked forward to using the machines, but soon learned that instructions must be listened to and carefully followed.

The one major drawback I found in using the machines was that of storage. Most existing classrooms would not have proper storage facilities; therefore, other tools for learning would vie with the machines for space.

I feel that the machines have a place in the curriculum as they would create a great desire to learn arithmetic better".

Teacher

"I am very happy to list some of the reactions of both students and teachers relative to our participation in the calculating machine experiment:

1. Pupils had fun using machines.
2. Greater interest in doing arithmetic.
3. Carrying was better understood.
4. Pupils found out what tables they didn't know.
5. Pupils realized the machines did only two processes—addition and subtraction.
6. Realized that multiplication was adding a number a certain number of times—machine recorded.

7. Realized that division was subtracting divisor from dividend as many times as possible—machine recording the number of times.
8. Enjoyed using machine to check their written work.
9. Enjoyed setting up answer on machine and check with their written work.
10. Value of machines was realized by figuring cost of thirty-three machines loaned.
11. Place value was better understood.
12. Pupils realized that machines must be operated accurately in order to be of value in checking work.
13. Pupils found that a mistake made in place value in addition in their written work could not be discovered if same mistake in value was made on the machine.
14. Proper placement of decimal point in money problems was emphasized".

Teacher

"In reply to your request for my reaction to the use of the machines, I feel that they were worthwhile in many respects. They proved a challenge to the pupils, and revealed their ability to grasp the technicalities involved to operate the machines skillfully. It made place value more meaningful and resulted in a higher rate of accuracy on daily assignments".

You might be interested to have the reactions of several of my pupils:

"They proved a help on checking work".
"They made us think faster".
"They made me feel capable of running a machine".
"It proved an easy way to check pencil and paper work".
"It gave me a sense of pleasure just to work a machine".
"It made me want to improve my arithmetic skills".
"It gave the teacher extra time while we checked our own work".

Teacher

"The children through the use of the machines learned the placement of numbers very quickly. Interest was stimulated and made arithmetic more enjoyable".

The comments made by the children after using the machines were:

"They made arithmetic fun".
"Helped us in learning our times tables".
"Helped us in long division, especially two-figures divisors".
"They help us in carrying".
"The machines were easy to use and we will know how to use the machines in later life".

Teacher

EDITOR'S NOTE. Here is an Experiment that does not suggest replacing paper and pencil computation with computing machines in our schools. Rather, it seeks to determine whether or not pupils using machines can and will learn to compute by both methods in the same amount of time as normally allotted to arithmetic of the conventional type. The gains in favor of the experimental group were small but significant at the 0.95 level of confidence. The 4½ month period should be sufficient so that the "joy-novelty" of using a machine should not be a determining factor. The experiment set out to determine growth in reasoning as well as in skill in computation. The editor feels that the test used is not an adequate measure of the various types and kinds of arithmetic reasoning we expect from a modern program in arithmetic. He believes that other experiments of this type should be carried out so that school people may know more about such items as: (a) levels at which computing machines are most worthwhile; (b) the several aspects of arithmetic for which machines offer aid in learning; (c) ways in which machines can be used to increase interest in and learning of basic mathematical understandings as well as the operations of arithmetic; and (d) modes of teaching and learning so that results will have greater permanence.

We live in a machine age. Many adults have access to a computing machine and have learned to place their faith in machine results. Let us continue to seek the optimum relationship of paper and pencil arithmetic to mental arithmetic to machine arithmetic.

A Teacher Plans Her Day*

MARY E. ALBRECHT
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I CONSIDER IT A PLEASURE and an honor to talk to a group of prospective and beginning teachers. Most of my time is spent in working with teachers who have taught less than five years. You have chosen a profession that not only gives you a great personal satisfaction and a livelihood but it also offers you the opportunity to work with all kinds of people, to develop leadership, to develop ideals, to stimulate ambitions, and best of all, to mold the character of our future citizens.

Let's think of those in our classrooms as real people. As new teachers we sometimes get so involved with courses of study and subject matter that we lose sight of the children. We should never talk down to children or lose our sense of humor. Always be able to laugh with your children; to learn; to grow; and to enjoy experiences together. We must love children and because we love them we want our classrooms to be the happiest, busiest, most challenging, stimulating and attractive place to be each day.

Each child needs:

1. To feel he belongs
2. To have a chance to take part in making plans and carry out suggestions
3. To feel that the other members of the group are his friends
4. To be given the opportunity to work out solutions to many of his own problems
5. To feel that what he does is important and is a real contribution to the group
6. To receive praise where it is justly due
7. To learn to appreciate the contributions of others; to be respectful and courteous to classmates and adults.

I am reminded of a little boy I once had in the third grade. He came to us from Chicago where his parents and older brother lived. His grandmother ran a rooming-house in our school community and he came to make his home with her. Let's call him Jack. Jack disobeyed all the rules both in and out of school. He never finished his assigned work unless kept after school and then it would be done and done correctly in a very short time. He went down the aisle and pulled threads out of a piece of weaving, that was almost finished. Then he would say he didn't know why he did it. Of course, he got into fights on the playground. When kept after school he was a delightful child with whom to talk. I found he had plenty of ability but he wanted the attention which only his misconduct seemed to bring. In talking about his parents and a brother in Chicago, I found out much about his family and his feelings toward them. After our social worker had investigated his previous school history we found the grandmother had brought him to live with her at a time when the school officials in Chicago had had him tested and with the approval of the parents were going to send him to a correctional school. It was also learned that the parents did not want the

* Given for prospective and new teachers at the meeting of N.C.T.M. in Milwaukee, April 14, 1956.

child and he was aware of this. There had been a serious rift between the parents when he was born. I called the grandmother in many times for conferences on how we could work together to help meet the needs of the boy. Following these talks he would be better for a few days but it never lasted very long. The grandmother said to me one day, "You are so patient and kind to Jack I just don't know what more we can do for him." She also told me that as she talked with him about his conduct and asked him why he caused his teacher so much trouble he had replied, "But, grandmother she loves me." I had never really told the boy that I liked him but there was something I liked very much about the child. This was an experience that taught me so clearly that in spite in all the correction he took, he knew instinctively that he was loved, and that is what he was striving to get from some one. That's what he wanted from his parents and did not get. I shall never forget this boy and what I learned from him.

Sometime later when our schools visited industry, as we do one day each school year, it was my good fortune to visit Ely Lilly and Company in Indianapolis. As many of you know, much of the country's medicine is made at the Lilly Company. I saw girls sorting little colored capsules, hour after hour, as they were electrically fed over a belt and defective ones would be pushed aside. They were paid by the hour to do this. I saw a man stirring big vats of salve of some sort. Still other huge bins of various coated pills would be tumbling over and over in the coating process; every one looking just alike. Girls sat for hours pasting very tiny labels on bottles; —the monotony of mass production. The sameness offered no challenge to thought. There was no opportunity to work with people except as fellow employees. It was after that visit to Lillys' that I thought to myself, "Thank God for children like Jack, he's different." I had appreciated him for the first time.

I tell you this experience because we must know our children before we can really do any planning for them. Our plans must be flexible enough in order that we may change and take another course when we learn it is a better one in the light of some added knowledge.

Teaching Is Complex

No one human endeavor is more complex than teaching because of its many intangible factors. By the way, Dr. John R. Clark has an excellent article in a recent issue (March 1956) THE ARITHMETIC TEACHER, published by the National Council of Teachers of Mathematics on the "Intangibles of Arithmetic Learning." In this article Dr. Clark stresses the importance of children experiencing success in learning. He says, "A child may forget his facts—but the way he feels about a subject lives on with him."

One must know his goals before attempting planning. There are some teachers that might say, "Oh, I don't plan very much and I get along as well as anyone." If that is the case just stop to think what could be accomplished with more planning. The possibilities are infinite. Briefly let me give you eight points or reasons for good planning:

1. Most recognized authorities agree that planning gives the teacher a clearer comprehension of the objectives and the relationship of her teaching to those objectives
2. It helps the teacher clarify her thinking
3. Planning tends to insure that the relative values of various instructional materials are given proper consideration
4. Planning reduces the amount of trial and error in teaching through a better organization of curricular materials; a greater economy of time
5. It also wins respect of pupils
6. Planning contributes to the teachers feeling of self-confidence and self-assurance.

7. Planning presents one of the teacher's best opportunities for continuous personal and professional growth
8. Finally, planning aids the teacher in recapturing waning enthusiasms thereby insuring a fresh, up-to-date presentation of instructional materials.

I believe most of our school systems are past the time of formally prepared lesson plans; of overemphasis upon form of the plan or that they be made according to certain required specifications in order to help the supervisors check up on the work of the teacher.

We need to plan with other teachers in our school or groups of teachers within a large city system. Teacher-pupil planning is necessary to let the children have an opportunity to express their ideas and try out their plans under guidance of the teacher. We can't say "do what you please, children." But we need to develop and teach values. To me, permanent learnings of real value, and growth in *self*-discipline are dependent upon the kind of pupil-teacher planning that is done. Such learning is not found in books; it comes only through experience.

I hope the day isn't far distant when the teacher who has a good idea to try out and it is educationally sound, will have a principal or supervisor who will say, "That's fine. Go ahead and try it. I'll be interested in your results" instead of saying "That isn't the way we do it in our school." Then of course, each teacher must learn to do her individual planning.

There is also need on the part of the teacher for pre-planning before approaching planning situations with the children.

1. The teacher must anticipate problems
2. She identifies objectives
3. She explores possibilities for learning experiences such as field trips and films
4. She explores sources of material such as found in children's museum exhibits, teacher's special library, state library, or the Chamber of Commerce
5. She increases her own background of knowledge through travel, professional reading and other means
6. She considers ways of organizing the class for work as in interest groups, ability groups or committees
7. She may wish to talk over plans with the principal
8. She might seek help from the supervisor or consultant
9. Or she may even wish to talk it over informally with parents

Planning Is Important

Each year as I work with large numbers of beginning teachers, I find lack of organization one of the biggest problems in the elementary classroom. Of course, most discipline problems stem from lack of organization. The orderliness, attractiveness and physical makeup of the classroom lend so much to the smoothness of the day's work and also to the creativeness of those in the classroom. If a teacher has a little corner where children can find scissors, paste, paper, and a box of colored scrap paper, it's surprising what creative results might develop for such materials being out where children can see them and where they are free to use them. This is a splendid way to encourage the use of manipulative materials. Another way is to have a table with a couple of dry-cells; a light bulb; a bell; a knife switch and a little wire, or even a lady-bug on a weed placed in a magnifying bowl for observation. The children can keep a diary of what they observe or make designs suggested by what they individually see. The possibilities are unlimited. Always have something alive or something growing in your classroom. Children have a great deal of fun with iron filings, paperclips, thumb tacks, rubber bands, a piece of glass and a few magnets. Much learning can come out of such interest centers.

In thinking of organization, I should like to tell you of my visit of a second

grade room recently. Shortly after entering the room the teacher said, "We are going to physical education now but when we come back we are having mothers in to hear a tape recording of some of the children's songs and poems. Won't you join us?" I have worked with this teacher for over a year and I assured her I would be glad to hear it as it was their first attempt to record. As she was leaving the room I noticed the boards had not been washed and several assignments had been partially erased. I found her bucket and some small pieces of sponge. One of the children who remained in the room, ran to get me a cloth from the janitor. Since this room was near the Principal's office he saw me washing boards and came in to inquire. He did not know that parents had been invited although he is very informal and it was quite all right with him. He told me later that he was embarrassed over the situation. Some parents arrived before the children came back and I welcomed them in to the room. Twelve or fifteen parents had accepted the invitation and sat at the tables with their children to listen to the tape recording. After the recording was over the teacher pulled out two big shopping bags—she had brought refreshments—it was to be a real party. She had provided paper cups and napkins and several large cans of pineapple juice and two large boxes of iced rabbit cookies which she had baked herself the night before. She quickly handed the napkins to two children who happened to be close by and began opening cans of juice. I immediately asked if I might help. She replied, "Oh, yes. Please do. I am so nervous. I'm glad you are here to help me." I jumped in to open cans, fill cups, and serve the juice and cookies. In a conference later, with this teacher, it was pointed out what a wonderful opporunity she had missed in not planning all the details with her children before hand and in letting them receive the guests and take part in the serving. They could have gone to the cooking room or brought trays and pitchers up which would have made serving so much easier and much more organized. There was so much social learning that had been missed. Of course, in her rush she never did notice that the boards had been washed. All the nice things for a party had been provided but the organization was entirely missing. She had been so generous as to provide a box of cookies for the class next door. Certainly it would have been a courtesy to the principal to have told him of her plans before she invited the guests and to have invited him at that time, or better still to have made a language lesson around the written invitation. However, she did ask him at the last moment. Whenever we invite parents into our rooms let's be sure the principal knows about it and also be sure our rooms are in perfect order including our chalkboards. It is just good public relations. We wouldn't think of inviting guests into our home and not have the details of the meal and how we were to serve it, well planned. So much is taught through example. Feel free to talk things over with your consultant or principal. Many of just such mistakes can be avoided and very enjoyable learning experience can be had by all when properly organized.

When we think of any particular school day let us always start it off together with our entire class on a high note of inspiration and spirtuality. Most of us use a patriotic or seasonal song with the Pledge of Allegiance, poems, and some scripture. Give it enough variety that it never gets monotonous. The teacher does not always have to plan opening exercises but in the older grades let different children plan the morning exercises for a week at a time, with the teacher's approval.

With younger children alternate group recitation periods with relaxation exercises or a song by the entire room. Do not keep little children at their seats too long at a time. For example, after individual read-

ing groups bring the entire class together for a game, music or poetry and dramatization. I found a plan that worked very well for me in a third grade to help provide the frequent change needed. I divided the room into three groups and rotated these groups. For example, while one group read with me in the chairs at the front of the room another group worked at their seats on some follow-up work relating to social studies, and the third group went to the chalkboard to write original stories using spelling words of new vocabulary words from a seasonal chart. These were flexible groups which changed from time to time as the children progressed.

The Math Class

Every math class should include

1. Review of concepts already learned
2. Motivation for teaching a new concept
3. A check-up on what has been learned in order to determine how much reteaching is necessary

In review of concepts already learned give some quick mental computation and use flash cards for concepts already established. If preparing to teach a concept from ones' to tens' place then give a review of adding two-place numbers without carrying. The important thing is to keep the sequence of learning logical.

Motivation is an important phase of a math class. To teach the concept of carrying you might add the lunch money and the milk money or you might have the children bring in newspaper ads and do some shopping. A toy store with toys priced or a grocery store set up in the room. The use of concrete and manipulative objects is very important and necessary to basic understanding. In this instance make use of real money to discover values. A placement chart is necessary in letting children handle objects to actually discover why we call one column ones, another tens, and another hundreds. Very little reteaching will be necessary if we stay with concrete objects long enough. There isn't a classroom, no matter what the level, that would not benefit by an arithmetic center in the room where children of varying degrees of progress can go to handle concrete objects and discover for themselves. It should contain various counting devices; beads, buttons, milk-bottle tops, disks, blocks or any object obtainable for counting or grouping. Many of these same things can be used to understand the meaning of multiplying and dividing. This center should always include measuring devices and a flannel board with fractional parts or other flannel objects of interest to children. Many arithmetic games can be made by the children or commercial ones obtained through which accurateness and speed can be developed. One game we made which the children always enjoy is the fishing magnet game. We cut fish out of various bright colored papers and put a number or combination on each one with a paper clip fastened in the mouth. Two children took turns playing with the fishing poles and magnets to see which fish would junp on his line. The child either wrote down the combination and answer or gave the answer orally. After I had made or obtained several arithmetic games and we learned to play each one, I found it very profitable and interesting to take a half hour each week and let the children choose the game they wished to play. We had from two to four in a group depending on the game. Each group kept score and I circulated among the groups. Such games, in which two or more children work together, not only develop concepts or learnings, but leads to growth in social relationships. Of course, no new concept would be taught in this way.

There is a difference between practice and drill. Give the children every opportunity for practice both at the board and on paper. While some work at the chalk-

board others can work at their seats with paper and pencils. I wish teachers would use their chalkboards more. Mistakes can be spotted and corrected so much more quickly at the board. We can all make good use of our supplementary math books for additional practice on specific concepts. After the concept is understood, *then* we can drill for mastery.

Our last step in the math class is checking upon the specific learning, or testing. Many of our math books give us tests in the back of the book or in separate booklets over every concept taught. In most cases the author has given permission for these to be duplicated for the teacher's use with the class. If this is not available you can make up your own test over the material covered and thereby determine those who need additional help.

We know that all children do not mature at the same rate. This brings us to the grouping of our class. Usually grouping comes under grade level or common need. These groupings should be flexible so that a child moves from one group to another as soon as he has learned the concept. This kind of grouping motivates learning.

We should constantly evaluate our teaching methods and our success in helping children to grow. May I tell you about another teacher? During President Ike's illness a little boy, with the help of his parents had written a "Get-well" letter and had received a reply from Mamie. When he brought it to school to share it with the teacher and the class, the teacher said, "Put it on my desk and don't forget to take it home with you when you go." When he arrived home the parents asked him what the teacher said about his letter. The child said, "Oh, she didn't have time to look at it and told me to put it on her desk and take it home." What kind of a teacher was that? You wonder about her. Of course, she could have been a democrat. Whatever you are, don't forget to live with your children and share and enjoy things together.

EDITOR'S NOTE. Miss Albrecht offers good advice to young teachers and to their helpers, the older teachers and school supervisers and principals. Many schools are experiencing an adjustment problem with the teaching staff due to "turnover" and with the influx of inexperienced teachers and those who have been away from teaching for some years. We must constantly remember that our schools are for the children and give to those who need it the necessary help in adjusting to our educational program. Kindliness, humility, and understanding are qualities found in good teachers. Frequently the beginners and returnees need help in the subject matter in order to bring to a class a teacher's depth of knowledge.

An Arithmetic Fair

The arithmetic classes of the Mountain Grove Elementary School served as hosts to high school mathematics students, teachers, supervisors and parents at an Arithmetic Fair, February 9 and 10, 1956. The elementary supervisor and elementary teachers sponsored the event.

A variety of student-prepared materials was exhibited. Instructional materials used in grade one through grade six, including manipulative, pictorial and projection materials were displayed. An enrichment program for arithmetic for grade three through grade six was an interesting part of the exhibit.

First grade pupils sang counting songs and second graders served as ushers. Interesting features of the fair were demonstrations given by pupils on recording numbers on the abacus, dividing and multiplying simple fractions, concretely, and the use of projection materials. The bulletin board that showed how the numerals were written at various intervals from 450 A.D. to the present time attracted attention.

The event seemed to be successful particularly in terms of arithmetical interest created in teacher, student and parent groups.

Reported by ELVA DUNN
Supervisor

Arithmetic at the School Camp

O. L. Davis, Jr.
Peabody Demonstration School, Nashville, Tenn.

Educators have long accepted as valid the principle that people learn best those things which they live. As teachers have tried to implement this basic belief, they have used various kinds of excursions, audio-visual aids, manipulative devices, and demonstrations in order that pupils may understand better what they are studying. In recent years, many schools have become more outdoors-minded and have taken classes to a school camp for varying periods of time. This school camping movement seems to be growing and many feel it serves an important function in making the world more real to the child who attends. By broadening and enriching his experiential background, the school camp provides many necessary situations out of which the pupil develops adequate meanings for himself.

As schools have taken children to camp, they have emphasized learning about nature and conservation. Schools have been cognizant, also, of the valuable contributions camping has to offer in the areas of social, emotional, and physical development of the pupils. The social studies, language arts, and arithmetic have been a part of the camping program rather incidentally, and often, accidentally.

This past year, desiring to maintain a strong program concerned with nature, i.e. animals, birds, weather, soil, etc., and conservation, the author resolved to make it possible for the pupils of his class to derive benefits from the camp environment which would aid their understanding of other areas of study.[1] This paper relates these efforts to the field of arithmetic.

Plans for the camp were developed in accord with a basic tenet of outdoor education: "that which ought and can best be taught inside the schoolrooms should there be taught, and that which can best be learned through experience dealing directly with native materials and life situations outside the school should there be learned."[2] Keeping this in mind, the arithmetic part of the program took two routes: a series of planned experiences at camp expressly devoted to an increased understanding in arithmetic was developed; and, realizing that many camp activities would provide opportunity for practice and enrichment in arithmetic, it was decided that they would be utilized when detected.

Measurement

Experiences in measurement of distance, height, and area when planned for a scheduled block of time at camp. These activities were chosen because they fitted most adequately an observed need of most of the class members, the basic philosophy of outdoor education, and the physical facilities of the camp. The 32-member class was divided into four groups, each meeting once early in the camp period for a two and one-half hour session.

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[1] The eighth grade at Peabody Demonstration School has gone to the school camp for the past eight years as an integral part of its year's program. The camp is a 67-acre college-owned plot, heavily forested and bordered on one side by a small river. With its cabins and deep water well, it provides an excellent opportunity for living and learning in the outdoors.

[2] L. B. Sharp, "Outside the Classroom," *The Educational Forum*, VII (May, 1943), pp. 363-364.

Distance

"How far is it from this maple to the pine over there?" Such a question opened the period. From the "Who cares?" to "About 30 feet," each pupil had a reaction to the problem as posed. Those who did not know how to estimate a short distance learned quickly to compare segments of the unknown distance with thoroughly familar lengths such as one's own height. "How do you know how close you are?" "Step it off." "But how long is a step?" Then, each pupil had his step measured. Stepping the distance and multiplying the number of steps by the length of his own step, he arrived at a result which was a check on his previous estimation.

"Joe got 33 feet." "I got 27 feet." "I got 30 feet even." "Why is it that we all don't get the same answer?" This confusion made possible a discussion of two fundamental concepts in measurement: the choice of unit and the constancy of the accepted unit. The step had been chosen by the group as the most conveniently obtainable unit and it fit the need for this linear measurement. The group readily saw that no two individual's "steps" were the same, also, that everyone's step varied from time to time. This inaccuracy was understood and accepted as a limiting factor in the usefulness of the unit. Another element contributing to inaccuracy was the conversion of a "step" into "feet," since one changed and the other remained constant. Thus, the pupils developed a concern for accuracy and for the proper choice of the unit of measurement.

Since it had no length of measuring tape, the group decided to continue using the "step" as a unit of measurement, consciously attempting to diminish such variable factors as the individual step and terrain. The children observed that they were reliving the experiences of men of a by-gone age in the history of mathematics.

"How can we determine the distance between these two trees without actually measuring it or stepping it off?" began another phase of measurement. Recalling classroom work with similar triangles provided the background for the solution of the problem. Using different people to step agreed upon distances and to stand at the points of the triangles, an answer was computed. Then, the procedure was repeated, invariably with a different result. "Why is this?" "Because the same person did not step all the distances," someone would volunteer and the importance of the constancy of the accepted unit again would be reinforced. Several times, pupils suggested the Pythagorean theorem instead of similar triangles as a method of finding the unknown distance. Two sides of the right triangle would be stepped off and the solution calculated. Comparison of the results of both methods proved very similar when the same person "stepped off" both situations.

At the river bank, the group set up similar triangles to determine the width of the river. By this time, practice enabled the pupils to estimate the distance fairly closely in relation to the solution obtained by indirect measurement.

Simple geometry, the use of square roots, and other classroom learnings became better understood as the children actually saw these techniques helping to solve problems which had meaning to them.

Height

Prior to coming to camp, the class studied the use of similar triangles in determining the height of an object such as a house and tree. It also studied the use of a Biltmore Stick, a device used by foresters for determining the number of sawable logs in a standing tree. During a period before supper one evening, the children took pieces of lumber $\frac{3}{8}'' \times 1\frac{1}{2}'' \times 36''$ and sanded them as the initial step in making a Biltmore Stick. During the scheduled measurement session, the Stick was calibrated, using units determined

from the standard Biltmore formula.[3] With a Biltmore Stick, the diameter of a tree at breast height and its height can be measured. The class chose several particularly tall trees and practiced finding their height and DBH. Using an International Rule table which the State Forester had sent the class, the number of board feet in these trees was estimated. With prices of lumber secured before coming to camp, the value of these trees was estimated. The concept of board footage was convincingly brought home to them.

This practice in using the Biltmore Stick was valuable when the State Forester visited camp one afternoon and discussed forest management. He pointed out, during a convincing walk through the woods, that some of the trees were worthless to the forest community and to the owner because they were not producing marketable lumber and were killing out saplings that would. His relation of profit in management to forestry was very successful in motivating discussions of how much money various trees would sell for, how much money the camp was not making because of poor forest management, and what trees to plant after worthless ones were cut. By boring, the class was able to determine the age of trees and the rate of their growth, two important factors in forestry, without damaging the trees.

Area

Pausing in its fast-moving session, the group was asked how big an acre was. Several responded that an acre contained 43,560 square feet. When asked to use trees as boundary markers for enclosing an acre, the pupils almost always described an area one-fifth to one-tenth the correct size. By finding the square root of the area of one acre, they determined one set of dimensions. Other sets were also computed. Then, the group was asked to step off an acre.

"Where do we start?" "Which direction do we go?" The class had studied phases of Tennessee's history during the year and were aware of the confusion which exists in the state today resulting from indefinite and inaccurate land titles. To dramatize this situation, they were asked to use a bush in an abandoned field as the starting point of laying off an acre. Starting from the bush, volunteers set to the task of stepping off the number of feet on one side of the rectangular acre. As the "steppers" walked on and on, the pupils made some observations. "Don't you think they've gone too far?" "I'll bet they're mixed up on the count." When the "steppers" stopped and turned around, other volunteers were asked to check their "stepping" by walking the distance too. There was usually a difference between the distances.

After everyone had become involved in laying out the acre, a discussion was held. "An acre is sure bigger than I thought." "My grandfather has a 100 acre farm. Gosh, that's a lot." "I sure would hate to mow a lawn that big." The size of an acre had been learned in an unforgettable way and a part of this nation's heritage had also become real to them.

Summation of Measurement Session

During this two and half hour session, the group had walked over a large portion of the camp site. The children were tired, but their enthusiasm was very high. They had developed meanings for themselves out of real, yet planned, situations in which the problems were accepted as their own. They learned that there must be a commonly accepted, convenient, and arbitrary unit with which to measure accurately; that the unit must be standard and not subjected to perceptible change; and that estimation is a very useful process and a necessary adjunct to meas-

[3] The principles upon which a Biltmore Stick are based and its construction and use are described in most standard forestry books. One source is Donald Bruce and Francis X. Schumaker, *Forest Mensuration* (New York, 1942).

urement and problem solving. Each pupil *understood* that there must be *precision* in measurement.

The scientific method with which they had become familiar during the year had received further use. Other vital arithmetic understandings were derived: concepts of distance, height, and area; insights into the uses of arithmetic and geometry in measuring, estimating value, profit, and loss; and a realization that mathematics has played a significant role in the social history of the world.

Other Activities Involving Arithmetic

The school camp provided a host of activities in which arithmetic was used. Practice in processes as well as understandings were gained as children met the problems of planning for and living at camp. A description of some of these experiences which formed a part of the arithmetic program at the Peabody camp follow.

Pre-Camp Planning

Specific preparations for the Peabody school camp began early in the spring quarter. However, in the fall, children were encouraged to begin saving money for the camp fee. For some, this was a concern which involved budgeting allowances and wise planning.

The costs of the ten days at camp were a part of the total camp planning. The children helped budget the expenses of food, supplies, transportation, and incidentals and gained an insight into the magnitude of the enterprise. Although their individual budgets for a ten day period involved only a few dollars, a budget of over six hundred dollars had to be handled for the class at camp. Figuring costs of food which the group wanted to eat while at camp brought together concepts in nutrition and quantity purchase of goods. The children decided that they couldn't afford to have steaks and chops as often as they wanted, that there was a sound reason why hash and casseroles find their way to the family table so often. Not only did planning the camp budget with class give it practice in skills and important understandings, but also this planning was valuable public relations with parents; they saw that the extra cost of the camp was spent wisely, for the budget was not hidden.

The class decided to have a camp bank and candy store. The school believed that the campers' nutritional needs were best met by regular, well-planned meals. If children wanted "extra" candy, it was deemed advisable to provide both a time for and a limit to the number of candy bars to be eaten. The children planned the kinds and quantity of candy to buy and developed a store staff. To make purchases, a "checking account" was established by each camper with the camp bank which was staffed by a class-chosen group. "Checks" were designed and dittoed and methods and materials for bookkeeping were chosen and understood by all before departure for camp.

Sometimes a special interest of class numbers led to an unexpected use of arithmetic. Such an instance was the request by a group of "camera bugs" to have a dark-room at camp in order to develop and print their pictures there. Others expressed an interest in this activity and it was decided to go ahead with the project. One boy developed a questionnaire by which the number of people interested in the project was determined, the number of rolls of film to be taken, and the number of people who would like to have their film processed for them. Then, the quantity of photographic materials was estimated and a cost-price for processing a single roll of film determined. A method of payment for costs incurred was decided upon through the camp bank and "checks" for the use of these materials were written against individual accounts.

Activities at Camp

As well as conducting the camp store and bank, other activities at camp had a direct relationship to arithmetic. Some of them were: determining directions with

and without a compass, by day and night, on sunny and overcast days; estimating distances hiked and length of time involved; making weather observations with instruments like thermometers and hydrometers and recording data and calculations; measuring dimensions and making scale drawings of cabins; and planning kinds and quantity of food for an overnight hike in relation to a determined maximum cost per camper. Mapping a portion of the campsite proved an interesting experience for some. At the last camp, the class decided to replace an old, rundown outdoor fireplace with a new one as a camp improvement project. Dividing labor tasks, figuring ratio of cement to sand, estimating time to complete the job, and analyzing minor expenses of the project provided another opportunity to use arithmetic skills.

Examining fossils found on hikes and investigating the geologic periods in which they lived made clear to many a concept of the extent of historical time. That the age of the campsite could be estimated by studying fossil remains and rock strata was very enlightening. It was humbling to many to learn the length of time necessary for the formation of an inch of soil. On one hike, the class examined a painting of a cliff done by an aboriginal Indian many hundreds of years ago; that a work of art, exposed to the elements so long, could withstand the ravages of time so well was to some incomprehensible.

Summary and Conclusions

The school camp is an excellent laboratory in which children may develop adequate understandings in many areas. These concepts, arising out of the child's own experiences, are gained in the most desirable way, through living and doing. The school camp is a pupil-teacher community in which both must face and solve reasonably well the social realities of life.

However, camp is no mystical place where pupils automatically learn when placed in combination with sunshine, water, fresh air, and exercise. Major activities at camp must be planned as fully as those carried on in the classroom. Every activity in which the children engage is a learning experience to the extent that it is recognized and used for that purpose.

Most assuredly, arithmetic has a significant part to play in the achievement of a rich camp experience. Some activities can and should be planned for the development and nurture of basic understandings in this area. Every situation should be examined to determine its potential in increasing arithmetic meanings. Several specific ways in which arithmetic has contributed to the Peabody camping program have been described. These activities are of such a nature that they may be modified easily for use with different age groups. With careful consideration of the philosophy and objectives of the camp, of both school and pupils, other arithmetic-related activities can be developed. It is incumbent upon those planning school camps to do so.

Editor's Note. Mr. Davis is a supervising teacher of the eighth grade at the Peabody Demonstration School. It is apparent that he not only has a rich background to bring to an educational camp but also he knows the importance of planning so that his work with pupils is much more than amusement and busywork. Other teachers will note additional arithmetical experiences which are not listed in the article. In a camp situation, as in a normal classroom, the teacher should be the guide and leader. The way in which this leadership is used determines to a large extent the values to be gained from school camping. Where pupils have a "felt-concern" and participate in planning as well as execution of a project learning is more real and forgetting tends to be lessened. It was interesting to note how Mr. Davis used a circumstance to educate children in the role they play in the financial success of a family unit. It was also refreshing to see the clear and concise statement of the basic tenet of outdoor education because enthusiasts in outdoor education have tended to make exhorbitant claims which have not been supported in the experience of others.

Arithmetic via Television

I. A Report of the Pittsburgh Experiment

HELEN K. STRUEVE
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5-4-3-2-1; THE INDICATOR on the television screen ticks off the minutes before the arithmetic lesson begins. Then, comes the 30 seconds' warning. At this point the children in the fifth-grade classes of the participating schools[1] are ready and know that the lesson will begin momentarily. And now the television teacher is presenting the lesson of the day, the next in a series of sequential developmental lessons. Every morning, Monday through Friday, a lesson is presented from 11:05 to 11:30 over WQED, Pittsburgh's educational television station. A five minute period prior to the beginning of the lesson is used by the classroom teacher as a warm-up period. This may be in the nature of a review of number facts or a recall of the skill taught in the previous day's lesson. Following the television presentation, the teacher has fifteen minutes which she devotes to supplementing or implementing the lesson of the day.

During the television lesson the children eagerly follow the instruction and participate in the varied activities. They respond to the teacher as though she were actually in the classroom. Hands are raised to show that they have the right answer. A verbal response comes urging the teacher to "wait a minute." Someone chuckles and says, "This is fun." Stimulated by the television teacher they manipulate concrete objects, such as the contents of their fraction kits, as an aid to discovering meaning or arriving at generalizations. The teacher's challenge to the youngsters to pursue a certain interest or area is accepted. Evidence of this is in the numerous items which come into the studio—magnificent scrap books, original problems based on newspaper ads, number stories, graphs and charts, or floor plans showing the dimensions of the rooms in their own homes. It is a thrilling experience for the TV teacher when she receives such material. Up until that time she has only anticipated how the children reacted to the lesson. The work sent in is so fine that the teacher includes some of it in her future lessons. When samples are shown on the screen the thrilling experience is reversed; the children in the classroom glow with pride.

Many of us who wondered honestly how the teaching of a subject, such as arithmetic, by television was possible have discovered what a powerful aid television is. Television cannot do the complete job but what it can do is amazing. First of all, and perhaps of primary importance, is this. Through this medium, it is possible to extend a superior teacher into many, many classrooms, thus giving hundreds of children the benefit of skillful instruction. The regular classroom teacher, whether she be an experienced one or a novice, is secure in the knowledge that tomorrow's

[1] Involved in this demonstration are the children of twenty fifth-grade classes of sixteen schools in ten school districts, including Pittsburgh. These children have three television lessons a day: one in reading, one in arithmetic, and one in French. All of the participating schools agreed to use a given text in reading and one in arithmetic. The language course is conversational French. These programs are viewed regularly by children in numerous other schools, even though they are not a part of the organized demonstration.

lesson, and the next, and the next, will be building toward a full understanding of the major concepts in arithmetic. This does not mean that the classroom teacher's role is not significant. She remains the mainstay of her classroom. Children will always need the close contact with someone who cares, with someone who will nurture their growing insight and understanding, with someone who is aware of their special needs. Because the planning of the basic lesson is assumed by the television teacher, the regular classroom teacher is free to concentrate in her planning on what she can do for the child who has been ill, for the one who has not "caught on," or for the one who has superior ability. She will move among the youngsters during the telecast, assisting where necessary and observing them closely to ascertain their needs. Providing for individual differences is almost entirely in the hands of the regular classroom teacher. This is the heaviest part of her assignment, for teaching by television, because of the very nature of the medium, seems best geared to the average or normal group. To help the regular classroom teacher in this respect, an outline of lessons for a two-week period is sent to her from the educational department. In this outline the nature of each lesson is indicated as well as the material in the textbook which can be used to extend or supplement the TV lesson.

The strength of the program is seen, in part, in the continued enthusiasm of the children. The energetic response in the initial period was attributed in some degree to the novelty of learning by television. But, when after nine months the enthusiasm is still apparent, one must acknowledge that here is a holding power that must be recognized. When reports come in that children absent from school did not miss any of their TV lessons, because they followed them on the set at home, this power must be reckoned with.

What is this power? Part of it is the facility to get and hold attention. The student must give his attention to the lessons without any dilly-dallying. He must follow the instruction as it is presented, because the TV teacher will not take time out to call him to attention. And if he isn't watching, he may miss the most rewarding part of the lesson—the use of a fine visual impression to clinch the understanding. In television visualization is not only possible, it is the very keynote to this type of instruction. Through the use of visual means it is possible to establish a meaningful setting for the development of the number concept. The children can be transported to a camp site, to the soap box derby, or to the airport without leaving their desks. They can be taken to the Indianapolis Speedway (comparison of hundredths in speed records), to the department store for Christmas shopping (problem solving), to the super-market (estimation, changing a fraction of a cent to the next whole cent), to the corral (perimeter, area), or to the produce yards (understanding how foods are brought to the consumer). What easier way can one apply the social approach to number? Then too topics such as the history of number, fascinating as they are, become even more fascinating when they are presented on television, for the visual aspect enhances then to the full. No visual device is used without purpose. It must serve as a motivating force, add meaning to the child's experience with number, or provide the opportunity for discovery, or its use is not justified. It is in this area that the TV teacher has an advantage over the regular classroom teacher. She has the time, the materials, and the resources to make every lesson a "special" lesson.

Each lesson is thoroughly planned. It is a segment of a long-range plan to develop sequentially a basic skill or understanding. It will have variety. This helps to keep the children's interest high. However, this is true of good teaching anywhere, but the TV teacher must remember it. It is neces-

sary if the lesson is to keep moving at a proper pace. No lesson is considered ready for presentation until it moves with ease. The thread of continuity is so woven into the lesson that the goal is reached in a seemingly effortless manner. If a certain skill is difficult to present or develop, the whole staff—teacher, supervisor, producer, director, and the artists—all get busy and exploit every possibility of the medium to bring about a satisfactory result. How many times the results have been more than gratifying! The very physical limitations of the screen become a challenge to all concerned. Certainly, the teacher finds herself constantly revamping her techniques to suit this engaging medium. The regular classroom teacher does not have the time to devote hours to such preparation for a single twenty-five minute lesson. Indeed, it takes more than the hours of a regular school day to prepare any lesson presented over television. Because this is true, it can be safely stated that these children in the demonstration are being taught a more thoroughly prepared lesson day after day than if they were being taught in the traditional classroom manner.

Another of the strengths of television is its focal power. With the help of the director and the cameraman the teacher can be sure the child is seeing what she wants him to see. Nothing obscures the point under discussion. If the subtraction step in the division process is being studied, the cameraman will frame up exactly on the area under discussion. Every child is seeing exactly what the teacher wants him to see. If the teacher is manipulating objects, such as fractional parts or the thickets in the place-value chart, every child can follow the procedure completely; there is nothing to hide what he is endeavoring to see. Indeed, it is very much as though the teacher were teaching each child individually. The child feels this is for him and, though he doesn't say it, he feels it is for him alone. Passivity is at a minimum. During the class period, the children work diligently. One little girl commented at the close of a lesson, "Phew, I'm as tired as though I'd been out on that athletic field myself!"

Teaching arithmetic by television opens the way for better public understanding of present day teaching methods. It is not uncommon for a lay person to call the studio and express his interest and approval of what he has observed in the days' arithmetic lesson. Occasionally, the far-reaching effect of television is highlighted. One day a letter came from a mother expressing her gratitude for the television lessons. Her young daughter was homebound because of illness, but, thanks to television she was able to keep up with her work.

Television teaching can be a tremendous force in the way of in-service training for teachers. Observing the systematic development of a whole year's work by a skillful teacher should prove invaluable to the beginning teacher. Here she would see the practical application of the theory she absorbed during college days. This can be her opportunity to evaluate her own procedures and techniques.

Although this year's work is labeled a demonstration, everyone involved is interested in the progress the children have made. If general reaction is any indication of what to expect, we can look forward to a wholesome report.

II. A Course for Teachers and Parents

Irvin Brune
Iowa State Teachers College, Cedar Falls, Iowa.

Teaching via the medium of television means hard work. Apparently, though, the results compensate for the efforts. The following comments represent the views of one teacher who taught one telecourse.

More than 50 per cent of the elementary teachers in Iowa have had no more than two years of college education. Moreover, among the several levels of education, it is mostly the elementary teachers who return to the profession after protracted absences. Having left primarily because of family responsibilities, many of these teachers still find it next to impossible to obtain additional formal education. Duties at home continue to loom large.

Accordingly, The Iowa State Teachers College, where the writer teaches in the Department of Mathematics, maintains an extensive program of field services, including full-time consultants, demonstration teachers, extension courses on campus and at centers reaching to the ends of the state, correspondence courses, lessons by radio, and television courses.

However, those who re-enter the profession need much more than college credits. They need concepts, subject matter, ideas, and methods. They need to become learners again. They need to acquire contemporary knowledge of how pupils learn. With all their getting, these teachers have to get understanding. Memoriter methods, although once popular, mostly miss the mark. So, courses by television help to bring people up to date.

Moreover, even before certain publications appeared, parents have been wondering why Johnny can't read, why Johnny can't figure, and why Johnny can't spell. Indeed, many of the more candid parents have long been wondering why Johnny reads better, spells better, and figures better than Daddy and Mommy can.

So, besides teachers who desire to learn, there are many parents who crave to learn. Numerous children, furthermore, accustomed as they are to sitting cross-legged before the family TV receiver, also view educational television regularly.

These, then, are the people whom courses by television serve:

1. A relatively small group of students who enroll for college credit.
2. A somewhat larger group of students who do not register for credit. These people seek ideas for their own thinking.
3. A group of thousands who view the lessons for whatever stimulation they can get.
4. A group of children who watch because of their interest in school subjects.

The television course deals with the subject matter of arithmetic and how to teach it. Professors on campus and at extension centers regularly teach another version of this course. Still another version can be studied by correspondence. Three meetings weekly for 12 weeks comprise the plan on campus. As a telecourse, ten lecture-demonstrations constituted the lessons. The ten topics and three subtopics under each follow:

LESSON 1. Basic Concepts

1.1 Groups are Collections of Things.
1.2 Groups Can be Ordered With Respect to Size.
1.3 Counting Sets Up a One-to-One Correspondence Between Number Names and Group Sizes.

LESSON 2. The Decimal Number System

2.1 Using a Base Facilitates Reckoning.
2.2 Non-decimal Bases Could be Practical.

2.3 Place Value Rates Highly Among Man's Achievements.

LESSON 3. Quantitative Situations

3.1 Additive Situations Require the Actions of Putting Groups Together.
3.2 Subtractive Situations Involve the Action of Removing a Subgroup from a Group.
3.3 Quantitative Situations are Processed by Recognizing, by Counting, by Recalling and by Calculating.

LESSON 4. More Quantitative Situations

4.1 Multiplicative Situations Involve Putting Equal Groups Together.
4.2 Divisive Situations Imply the Removal of Equal Subgroups.
4.3 Multiplicative and Divisive Situations are Processed by Recognizing by Counting, by Recalling, or by Algorisms.

LESSON 5. Arithmetical Processing

5.1 Counting, Readiness, and Operating are Closely Related.
5.2 Discovering, Stating, and Practicing Facts.
5.3 Why Do We Operate As We Do?

LESSON 6. Common Fractions

6.1 Understanding Is the Secret.
6.2 What Do Fractions Mean?
6.3 Operations Follow Meanings.

LESSON 7. Decimal Fractions

7.1 Decimals Illustrate a Novel Notation for an Ancient Idea.
7.2 Judgment Underlies Placement.
7.3 Per Cents Are Fractions Too.

LESSON 8. Measurement

8.1 What Is Measurement?
8.2 How Standards Meet a Need.
8.3 Using Data with Cave.

LESSON 9. Problem Solving

9.1 What Is a Problem?
9.2 Mental Arithmetic Assumes a New Look.
9.3 Approximation and Estimation Foster Number Sense.

LESSON 10. Effective Teaching

10.1 First Comes the Idea: Then the Symbol.
10.2 Teaching is More Than Telling.
10.3 What Is the Teacher's Job?

All of the lessons were telecast (live) on Saturday mornings. The first six began at 8 and stopped at 9:30; the last four went from 7:30 to 9. Two five-minute breaks occured in every lesson, each after 25 minutes. That left 30 minutes for the third segment. Each of the three subtopics constituted the subject for a segment of time. The lecture-demonstrations had to fit the times precisely. There were no rehearsals.

Teaching by television entails drudgery. As seen from here now, the work constituted a full-time job. This holds despite the fact that the Division of Field Services at the Iowa State Teachers College handled all the publicity, registration, and secretarial work involved. Indeed, a competent shopman constructed all the visual materials needed. And, of course, the television station provided the technical staff and equipment essential to the telecasting itself.

The teacher's job included these items:

1. Planning the course.
2. Preparing the syllabus.
3. Choosing the visual aids.
4. Writing brief summaries of each lesson.
5. Planning the telecasts.
6. Reading two written lessons required of each student.
7. Determining grades.
8. Answering correspondence.
9. Teaching on campus.

Some Advantages of the Work

1. Television brings the course into the student's home. Many teachers are married; many have responsibilities at home and on the farm. But all can view lessons at home (or at a neighbor's home). All can read at home.

2. Economies of time and money enable more people to do college work. Travel time and living expenses discourage some people from taking needed work on campus.

3. Many people can improve themselves by doing the work even though they don't need collegiate credits.

4. Visual materials urgently needed in many classrooms receive the emphasis they should have. Television is the ideal medium for acquainting thousands of people with simple devices pupils can use to learn the big ideas in arithmetic.

5. Television also is ideal for acquainting thousands of people with recent findings about how pupils learn. How to utilize those findings in teaching arithmetic also becomes apparent. Especially can parents profit from such disclosures. They can thereafter appreciate why teachers guide learning in arithmetic as they do. Times have changed. Misconceived procedures formerly used now steadily lose their popularity.

Some Disadvantages of the Work

1. Learning inherent in discussions is lacking. Some students compensate for this by meeting with others in their home communities. Thus they can talk the lessons over, and learn thereby.

2. Most of the evaluation of the students has to come from the final examination. The instructor does not know his students. Some become nervous at examinations. Some might be ill at that particular time. Tests, fallible enough at best, can hardly measure a person accurately in one sitting.

3. The instructor lacks the reactions of his students to his teaching. In his classroom he gauges his efforts according to student responses. Via television he understands neither their comprehensions, nor their apprehensions, nor their misapprehensions. The camera permits only a one-way trip. It transmits actions remarkably well. It admits reactions and interactions not at all.

Some Vexing Questions

Stemming mostly from the apparent disadvantages of teaching by television, the following questions obtrude persistently:

1. How can we better evaluate the work of unseen students?
2. How can we pace our presentations to people we cannot observe?
3. How can we more adequately compensate for the learnings our students forego when they study televised courses?

The Professor Speaks

Here are the writer's opinions after his single stint:

1. Teaching by television teaches the teacher. It is different. One's material has to be condensed. To emphasize the essential and to delete the less vital requires careful planning. To read the written lessons (largely essay answers) is formidable. To answer student mail takes time and thought.

2. Teaching by television tends to better one's courses. Careful organizing eases the learner's task. Attempting to communicate with a mass audience makes a man mend his presentations. Condensing one's subject matter stresses key ideas.

3. Would the present instructor agree to teach another telecourse? The answer, and it has been much pondered, is yes. Despite the discomfort caused by not knowing how one's teaching affects one's students, this instructor would gladly try some work by television again.

III. Demonstration Lessons at Springfield

R. C. GLAZIER
Springfield Public Schools, Springfield, Mo.

Last spring the public schools of Springfield, Missouri presented two demonstration lessons in arithmetic. One of these was conducted by Mrs. Naomi Hall with her first graders at Sunshine School and the other by Mrs. Thelma Smith with a sixth grade from the same school.

Mrs. Hall demonstrated the development and structure of the number system with meaningful approaches for primary grade children. Mrs. Smith presented the use of Arithmetic skills in the functional situation of making a report on the school lunch. The need for drill for the retention of skills was illustrated.

The first grade demonstration included:

1. Review 10–20
2. Cards and other aids to develop understandings of 10–20
3. Lunch order as a functional experience
4. Drill on all processes
5. Seatwork with children busy at desks

The sixth grade lesson included:

1. Presentation of lunch report for the day
2. Briefing of new lunch managers
3. Showing checking of arithmetic
4. Drill on fractions

EDITOR'S NOTE. What will we do with this new and powerful educational medium of television? At this stage it is necessary and important that we experiment in order to learn what types of materials and what phases of the school program can be readily learned via TV. Certainly it is a fine avenue for informing parents and the public about the schools but is it a good medium for teaching arithmetic? How can we make a program pay big dividends in terms of good leaning and not become merely another passing entertainment? As Dr. Brune pointed out, it is hard work which requires a great deal of careful planning. Let us not be afraid to experiment, always keeping in mind that educational television should be truly educational.

Minutes of the Annual Business Session

HOTEL SCHROEDER, MILWAUKEE, WISCONSIN
APRIL 13, 1956

Mrs. Marie S. Wilcox, President, called the meeting to order at 4:30 P.M.

1. Miss Virginia Lee Pratt moved, seconded by Mr. Joseph Urbancek, that the minutes of the April 15, 1955, meeting be approved as printed. The motion carried.

2. Mr. M. H. Ahrendt, Executive Secretary, gave a preliminary financial report. He stated that the final report would not appear until after the close of the year which ends on June 1st. The completed report will be published. Mr. Ahrendt then reported on the new office arrangements, the new membership files and gave a brief description of his work over the past year.

3. Mr. Houston T. Karnes, Recording Secretary, gave a brief review of the actions of the Board during the past year. This review included items which were thought to be of interest to the members. The review follows:

(1) All official notices must be carried in both *The Mathematics Teacher* and THE ARITHMETIC TEACHER.

(2) THE ARITHEMTIC TEACHER, beginning with the 1955 October issue, would have six issues per year.

(3) Student Membership is now available to all full-time students.

(4) No meeting for Christmas 1957, is being scheduled. This is experimental in view of recommendations for only two meetings per year.

(5) The Curriculum Committee at the Secondary level has been given permanent status and will be increased to a total of fifteen members. Funds are to be secured for a major study.

(6) Future meetings: Summer 1956, Los Angeles, California; Christmas 1956, Jonesboro, Arkansas; Spring 1957, Philadelphia, Pennsylvania; Summer 1957, Northfield, Minnesots; Spring 1957, Cleveland, Ohio; Summer 1958, Greeley, Colorado; Spring 1959 or 1960, Buffalo, New York.

(7) Report of the Membership Committee shows a fine increase during the past year. Future plans of the Committee should result in an excellent continued growth.

(8) A Directory of the Membership of the Council is to be published. The names will be listed both in alphabetical order and by States.

(9) 1957 is the Centennial year for N.E.A. The Council is planning to cooperate in the celebration.

(10) Mrs. Wilcox announced to the Board that contacts had been made with the National Science Foundation in an effort to secure funds for:

a) A Writing Conference for the Yearbook on Concepts.

b) The Publication and Distribution of a Mathematical Guidance Leaflet. Although the final decision of the Foundation has not been given, the request was favorably received.

4. Mr. Maurice Hartung, Chairman of the Nomination and Election Committee, gave the following report of the recent balloting:

President: Mr. Howard Fehr
Vice-President (Senior High School): Mr. Donovan Johnson
Vice-President (Elementary): Mrs. Laura K. Eads
Directors: Mr. Philip Jones
Mr. Philip Peak
Mr. H. Vernon Price

5. On behalf of Mr. F. L. Wren, Chairman of the new Nomination and Election Committee, who was not able to attend due to illness, Mr. Maurice Hartung explained the procedure of the Committee and invited recommendations from the membership.

6. Mrs. Wilcox introduced the newly elected officers and directors and invited Mr. Fehr to address the meeting.

7. Mr. Fehr expressed his appreciation for the honor which had been bestowed upon him and asked for the full support of the membership in the great task that will be his during the next two years. He then spoke of the great progress the Council had made during the past ten years and of the Council's place and responsibility in the field of education.

8. There being no old business Mrs. Wilcox called for new business.

9. Mr. H. C. Christofferson moved, seconded by Mr. Anton S. Richert that the retiring president and the officers and directors be given a vote of thanks for the splendid job which had been done. The motion passed.

10. Mr. Philip Jones moved, seconded by Miss Lurnice Begnaud, that Miss Margaret Joseph and her committee members, be extended a vote of thanks for the excellent arrangements for the Milwaukee meeting. The motion passed.

11. There being no further business the meeting was duly adjourned at 4:55 P.M.

Respectfully submitted,
HOUSTON T. KARNES
Recording Secretary

The Controversy Regarding the Teaching of Higher-Decade Addition

FRANCES FLOURNOY
University of Texas, Austin, Texas

THERE IS CONSIDERABLE DISAGREEMENT regarding the teaching of higher-decade addition. It seems to be a question of both "when and how." A study of published research at this time does not reveal any studies which might help in solving the problem of when and how to teach higher-decade addition.

In higher-decade addition a one-figure number and a number with two or more figures, usually two figures, are added. Higher-decade addition which involves bridging is the more difficult type because the tens place in the sum is in the next higher decade than is the tens place in the two-figure number to which a one-figure number is being added. This is commonly called "bridging the tens."

In adding down in the column of figures to the left, the first higher-decade addition is 14+8 equals 22 and it involves bridging from teens to twenties. The next instances is 22+4 equals 26 but does not involve bridging. The last higher-decade addition is 26+7 equals 33 and involves bridging from twenties to thirties.

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 5
 9
 8
 4
+7
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Higher-decade addition may also occur when carrying in multiplication as in

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 59
×9
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It is necessary in this example to add 45 and the carried 8. This is higher-decade addition which involves bridging. In order to add 54 and 8, one must bridge from forties to fifties.

It is important that children be helped to gain skill in performing the higher-decade additions when involved in column addition and when carrying in multiplication. Though there seems to be little or no research to prove this to be the case, observation and experience in the classroom from Grade 3 upward readily reveals that higher-decade addition, especially that involving bridging, is very difficult for children to master. Many college students admit difficulty also and devise various ways of adding a column of figures in order to avoid higher-decade additions that may be involved.

The Problem

The questions of when and how higher-decade addition may most effectively be presented to children and be learned by children are considered by this writer to be significant in the teaching of arithmetic. The purpose of this article is not to suggest or try to prove one method as better than another. For immediate consideration, however, and as a part of this total problem, the following questions are being considered in this article:

1. How do present-day textbooks for children present higher-decade addition?
2. On what points do authorities in the field of arithmetic agree or disagree regarding the teaching of higher-decade addition?

Procedure

Six series of arithmetic textbooks published since 1950 were examined. In each of these, higher-decade addition was first introduced in Grade 3. Each Grade 3 book was studied in order to note the following:

1. In what printed form is the higher-decade addition example presented?
2. At what place in the textbook for Grade 3 is higher-decade addition with and without bridging first presented?
3. Is higher-decade addition presented through a column addition problem situation so that this peculiar need for this skill might be recognized by both the child and the teacher?
4. How is higher-decade addition without bridging presented?
5. How is higher-decade addition with bridging presented?
6. Are children given practice in adding with unseen numbers and without paper and pencil as is necessary when performing the addition involved within column addition and when carrying in multiplication?

The section on higher-decade addition was reviewed in each of five professional books on arithmetic published since 1950. Opinions regarding when and how to teach higher-decade addition were noted.

Findings

How do present-day textbooks for children present higher-decade addition?

1. Three of the six books examined use only the vertical printed form, $\begin{array}{r} 15 \\ +6, \\ \hline \end{array}$ for presenting the higher-decade addition example. The other three books use both the vertical and the horizontal forms, $(15+6=____)$. One of these three places special emphasis on the use of the horizontal form.
2. Higher-decade addition without bridging is usually introduced soon after the addition of two two-figure numbers with no carrying. Only one of the books introduces higher-decade addition before this point in the sequence of steps in learning to add. Two of the books introduce higher-decade addition with bridging before carrying in addition is introduced. One of the books introduces higher-decade addition with bridging after carrying has been taught. In three of the books carrying and higher-decade addition involving bridging are introduced simultaneously.
3. Five of the six books introduce higher-decade addition both with and without bridging apart from the column addition situation in which it is frequently needed. Several pages later in these five books attention is drawn to the need for higher-decade addition in column addition and practice in column addition is given. One of the six books introduces higher-decade addition in the column addition situation.
4. When higher-decade addition without bridging is introduced, all of the six textbooks propose teaching the method "adding by endings" which emphasizes the key basic fact ending. The basic fact ending is noted in several examples as

$$\begin{array}{rrr} 3 & 13 & 23 \\ +4 & +4 & +4. \\ \hline \end{array}$$

5. Three different methods are used for teaching higher-decade addition involving bridging. One of these methods is to add by endings and to bridge the tens. Another method is referred to by some as the "tens method." This is a method by which enough is taken from the smaller number and added to the larger number to make the larger number a tens number with no ones. Then the remainder of the smaller number is added. An example of this is $28+6=28+2+4=30+4=34$. A third method found to be taught is the same carrying method as is done when adding two two-place numbers.

Two of the six books make use of the "adding by endings and bridging the tens" method. Two of the books first use the carrying method and then introduce the "bridging the tens" method as a short cut. In only one book the carrying method is taught. In one other of the six books two methods are used. These are "adding by endings and bridging the tens" and "the tens method," as described in the preceding paragraph.

6. In one of the six books attention is given to helping children learn to visualize the number they must keep in mind but cannot see in the column of figures to be added. None of the six books provides systematic practice on higher-decade addition in "without paper-and-pencil" exercises. By "without paper-and-pencil exercises," the writer means higher-decade addition in which children see only the one-figure number, keep in mind a two-figure number given by the teacher, and respond orally without the aid of paper and pencil.

How much agreement is there among authorities in the field of arithmetic as to how and when higher-decade addition should be presented?

Morton[1] states that in higher-decade addition with bridging, just as without bridging, the pupil must see the relation of each higher-decade combination to a basic fact. He further states that in column addition a person competent in addition reacts mentally in a manner quite different from his manner of reaction in carrying. When adding 12 and 9 in a column of figures, it is suggested that the competent person would think 12, 21 and would not need to use or visualize the slower carrying method. He recognizes, however, that the pupil who is just beginning to learn higher-decade addition cannot react in such a highly efficient manner. The beginner will be slow and deliberate in his thinking but with the teacher's guidance pupils can become more skillful. Morton is very emphatic in his opinion that if carrying is taught before higher-decade addition involving bridging or if higher-decade addition is taught as an application of carrying, inefficient habits of thinking will be formed.

Brueckner and Grossnickle[2] in agreement with Morton that decade addition is not meaningful until the pupil has discovered the relation between the key fact and the ending in higher-decade addition. They emphasize that there should be no carrying as such in higher-decade addition involving bridging. They stress the use of this form, eighteen and $4 = ____$, first and state that the pupil is less likely to make two mental responses when this horizontal or equation form is used.

Wheat[3] in agreement with Morton and Brueckner and Grossnickle regarding the way in which higher-decade addition with and without bridging should be presented to children.

Spitzer[4] calls attention to the need for helping children discover the basic fact key ending in higher-decade addition without bridging. He points out, however, that we have little evidence to show the effectiveness of this method in higher-decade addition with bridging. It is his opinion that the tens method may be a helpful method for the child to learn when adding with bridging involved.

Stokes[5] suggests that adding by endings and bridging the tens be deferred until

[1] Robert L. Morton. *Teaching Children Arithmetic*, p. 150–52. New York: Silver-Burdett Co., 1953.

[2] Leo J. Brueckner and Foster E. Grossnickle, *Making Arithmetic Meaningful*. p. 246–47. Philadelphia; John C. Winston Co., 1951.

[3] Harry Wheat. *How to Teach Arithmetic*, p. 128. Evanston, Illinois: Row, Peterson and Co., 1951.

[4] Herbert F. Spitzer. *The Teaching of Arithmetic*, p. 114–15. Boston: Houghton Mifflin Co., 1954.

[5] C. Newton Stokes. *Teaching the Meanings of Arithmetic*, p. 136. New York: Appleton-Century-Crofts, Inc., 1951.

after Grade 3. He judges adding by tens to be a better method to be used at first. He states that adding by tens is a transition step between counting and the processes of adding by endings and bridging the tens.

Summary and Comments

This study of arithmetic textbooks and opinions of authorities in the field of arithmetic points up the presence of controversy or at least considerable lack of agreement regarding the teaching of higher-decade addition. There is definitely a lack of published research on the teaching of higher-decade addition. The textbook procedures now in use regarding the teaching of higher-decade addition and various opinions of arithmetic authorities on this subject do not seem to be supported by sufficient research.

The following statements are presented in summary:

1. All of the books examined first introduce higher-decade addition in Grade 3.
2. One-half of the textbooks examined use only the vertical form, $\begin{array}{r} 15 \\ +9 \\ \hline \end{array}$, for presenting higher-decade addition.
3. There is considerable variation in opinion and practice regarding the place at which higher-decade addition is presented in each book. Some introduce it before teaching carrying, some after carrying, and one introduces it simultaneously with carrying.
4. All the textbooks studied emphasize giving attention to the key basic fact ending when performing higher-decade addition.
5. Three methods for performing higher-decade addition involving bridging were found to be presented in texts. These are "adding by endings and bridging the tens," "the tens method," and "a carrying method." No textbook uses all three: some books use two. No authority, whose professional book was examined, advocates the use of the carrying method as such.
6. Most of the textbooks do not first present higher-decade addition in a setting such as column addition, in which it is needed. It is the opinion of this writer that, when first introduced in the textbook, there is an insufficient amount of textbook practice exercises with higher-decade addition as needed in column addition.

Many questions arise in the thinking of this writer regarding the teaching of higher-decade addition. Does the use of the horizontal or equation form (15+9 =_____) encourage the child to use a different pattern of thinking than does the vertical form? Does any of these methods of thinking, when bridging is involved, work more effectively for the child than another method in the column addition situation or when carrying in multiplication? To what degree can we expect third grade children to master the method of adding by endings and bridging the tens? Do textbooks make it clear to the children and to the teacher whether after noting the basic fact ending, the pupil should first add and record the ones and then record the tens or whether he should think of the whole answer almost in one complete thought? Do teachers understand "adding by endings" and "bridging the tens" as a different pattern of thinking than carrying from one column to another? When and how may higher-decade addition be most effectively presented to children and learned by them?

(Editor's Note on page 176)

The Wonderful World of Mathematics by Lancelot Hogben. 70 pp. Garden City Books, Garden City, New York, 1955. Price $2.95.

This book is a delight. In it, Professor Hogben tells "how the growth and development of civilization is also the story of the growth and development of mathematics as a science." Professor Hogben's skill in presenting specialized subject-matter to the layman is well-known to us through his *Mathematics for the Million*. In *The Wonderful World of Mathematics*, the text is coupled with superb pictures and maps in color, the publishers say over 250 of them. The page size is large and the arrangement of illustrations and text is varied and skillful.

The end papers are an integral part of the book. Here on a double spread is a summary of the text in pictures. These take you from the use of fingers to indicate numbers to counter reckoning and to numerals; from observing the Great Dipper to Galileo and his telescope; from the rope stretchers of Egypt to a surveyor with a theodolite; and, at the lower left, separated from the rest by a body of water, a Mayan developing his numerals in Central America. Spend some time with these pages before going to the book itself, and when you have finished the text, use the end papers for review. You will be surprised to find how much new material they contain.

The organization of the book is one of its strong points. The story is close knit. It moves fast. The treatment is chronological. The illustrated summary is excellent.

The book will be useful to teachers and to their pupils. It will intrigue the general reader. In the chapter on Egypt for instance, you see a pyramid under construction, the overwhelming number of men required to move the blocks of stone from the bank of the Nile where they had been brought from quarries up stream, the ramps over which the blocks were hauled to the working level. With this setting, you are eager to find what the author has to say about Egypt.

The style and color of the illustrations are varied and thoughtful. A picture indicating that in Greece music was a part of mathematics might almost have been taken from a Greek vase. The picture of a man and an electronic calculator is completely modern in drawing and color.

The picture describing the measurement of the diameter of the earth by Eratosthenes is excellent.

A book like this is not the place for the display of meticulous scholarship, but several things seem unfortunate.

In discussing the abacus, Dr. Hogben limits himself to beads in grooves. The artist has followed this with the result that a picture which seems to be based on an illustration in the *Margarita Philosophica* (1503) shows a computer working with marbles instead of the more convenient flat counters that appear in the original. It is unfortunate that Dr. Hogben chose to repeat the Galileo-Leaning Tower legend particularly when another cut shows Galileo's experimental apparatus. Another item to be regretted is the multiplication of 123 by 165 written elaborately in Roman numerals. The partial products are noted and then they are added. The result displays twenty M's to represent 20,000, suggesting that Roman numerals went no higher than the M for 1000. This example is not a part of the text proper, and it may be that Dr. Hogben is not responsible for it. One suspects that it has no basis in fact. The author states that "somehow they (the Egyptian surveyors) found that pegging out certain lengths of rope in the form of a triangle produces a

right angle opposite the longest side." The 3-4-5 triangle is then cited and also one with sides 5, 12, 13. There seems to be no evidence to support the statement that such relationships were used, yet, like the Galileo story it is repeated over and over again.

A drawback to making any suppositions no matter how carefully qualified, is that the reader tends to replace the supposition by making it a fact and ignores the provisos. A specific illustration will illustrate this point. After mentioning the Babylonian tablets that give multiplication tables and tables of squares of numbers, Dr. Hogben says "It seems likely that the priests of those days discovered a way of using square tables . . . to multiply any two numbers together. . . ." The method which follows uses the relation that the product of the sum and the difference of two numbers is the differences of their squares although this is not explicitly stated. The computer is supposed to find the average of the two numbers and the difference between the average and the numbers. The square of the difference is subtracted from the square of the average thus yielding the desired product. This method is ingenious and alluring. The author's statement has the proper safeguards. But to the reader the qualifying words tend to disappear and the "seems likely" may become an assertion. This is borne out by the review in *Time* Nov. 7, 1955 when the book was first released. The reviewer states that "By using tables of squared numbers, the Mesopotamians learned to multiply without the use of the abacus" and Dr. Hogben's illustrative example *102 multiplied by 96* follows. This likelihood of misinterpretation argues that in a work of this type, such conjectures should properly be omitted.

The Wonderful World of Mathematics will be widely used by pupils and by adults. It is a well-told story in a fine setting. It is too bad that it has these flaws.

VERA SANFORD

Arithmetic at Work—Book 1 ($2.80) and *Arithmetic in Life*—Book 2 ($3.00) by Howard F. Fehr and Veryl Schult. D. C. Heath and Company, Boston.

It is the intention of the authors of the above-named arithmetic series to offer a modern, well-organized arithmetic program to meet the needs of Grades 7 and 8. To accomplish this, all topics necessary to provide a complete, balanced course of study have been adequately covered in these books.

A point well to be considered is the attractive, colorful appearance of these texts. The many well-arranged diagrams and illustrations and the large, easily-read print offer special inducement for creating immediate pupil interest.

The broad scope of these books is one of their fine points inasmuch as it allows the teacher a great deal of flexibility in planning a program for both the bright and the slow pupils in his class. One topic which might be considered enrichment for the brighter pupils to stimulate and challenge their thinking is covered in Chapter 15 of Book 1, where advanced concepts of large and small numbers in relation to the atomic age and the use of precision instruments are introduced. In Book 2, Chapter 14, the metric system could be used for a similar purpose.

Another excellent feature of the texts is the fact that so much of the material is directly related to the everday experiences of the pupils. Extensive coverage of consumer information on insurance, savings, wise spending, and investment provides real practical training for all future wage earners.

The innovation of presenting, in the introductory chapters, relatively new materials on measurement, a topic close to the lives of all pupils, would seem to be a commendable procedure, since they would have immediate appeal and arouse interest. These are then followed by chapters devoted to the review of the fundamental processes for the purpose of gaining skill in computation.

In considering the general nature of these texts, it should perhaps be mentioned that a characteristic common to both books is that, because the explanations lead so quickly into rules of "how-to-do-it," there seems to be the assumption that the teacher will necessarily have to do a great deal of development of the particular topics to make the rules meaningful rather than mechanical. On page 274 of Book 2, a rule is given for equal fractions, the mechanics of which is easily followed, but the meaning is not clear.

One page 48 of Book 2, the title "Meaning of Fractions" suggests only one meaning whereas in Book 1 a corresponding title "Meanings of Fractions" indicates more than one. This might be confusing in developing a true understanding of fractions. It is unfortunate that the text refers to "disc" and the illustration uses squares.

As in any set of books, particularly in the first printing, these arithmetics have statements which may be misleading to pupils or which need more explanation than is furnished. For example, on page 65 of Book 1, the 3 in "$11 \div 3$" is not apparent; one page 91, the illustration of a *circle* and the word circle suggest that the circle is the space within the curve; on page 105, a part of the pie seems to be missing; on pages 117 and 136, one finds some rather peculiar fractions to be added which are good for brighter children to use in applying basic principles and for that purpose probably should be set apart or placed near the end of the section; and in a number of problems the information furnished is not according to common usage and sometimes is not factual. Certain pupils like to believe textbooks and when one reads that the boiling point of alcohol is 175° he is apt to accept this as correct.

On page 226 of Book 2, it appears that somewhat misleading information is given in regard to Christmas or Vacation Savings Plans earning interest. This may be true of a particular bank but is not a general practice in many areas.

For the benefit of both pupil and teacher a vocabulary list, summary, chapter review, and chapter test are found at the conclusion of each chapter. In addition, cumulative reviews and tests are included at regular intervals. A Teacher's Manual which should be of great help to the teacher is also provided.

In conclusion, we feel that these modern textbooks compare favorably with any published today and that they would enrich the arithmetic program of most Junior High Schools.

MICHAEL GANCE
KATHERINE F. LUCE

Flournoy—Higher Decade Addition

(*Continued from page 173*)

EDITOR'S NOTE. Miss Flournoy has investigated textbook treatment of one of the technical elements in learning addition. Some children easily learn to go from a basic addition combination to one in the decades and even "bridging" seems no great difficulty. Other children require considerable practice. Here is one of the spots where specific practice is often useful. For example, a blackboard drill might have decade numbers placed around a circle and a digit placed at the center, then as the teacher points to one of the numbers around the circle the pupil adds the digit at the center mentally and states only his answer. This and other practice devices can be prepared so that "Bridging" is included or avoided. Since in actual column addition the number to which the next digit is added is a mental or unseen number a more advanced procedure is used for further practice. This involves writing on the board a series of one-digit numbers which are then added to a decade number spoken by the teacher. When such a drill is used in connection with multiplication it should be remembered that the "carry figure" is always less than the digit used in the multiplier. For example when multiplying by 7, the "carry figure" never exceeds 6. Let us help our pupils to learn to compute with facility and accuracy.